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OPTIMIZING PARALLEL OPERATION OF AGGREGATES EQUIPPED WITH «CROSSFLOW» TURBINES

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Abstract

The paper addresses the issue of optimizing the parallel operation of two identical hydropower aggregates equipped with Crossflow turbines under variable water flow conditions. The objective of the study is to determine the optimal distribution of the plant total flow between the aggregates that maximizes the capacity at a given flow rate. The optimization is formulated using the method of Lagrange multipliers, with the objective function proportional to the hydraulic capacity under constant head conditions.

It is shown that when the plant flow does not exceed the nominal flow of one aggregate, the maximum capacity is achieved by operating only one turbine, and no dispatching control is required. In the range of parallel operation of two aggregates, a nontrivial optimization problem arises. Numerical analysis based on manufacturer performance curves of Crossflow turbines demonstrates that the maximum total capacity in this operating zone is achieved when the flow is evenly distributed between the two aggregates. The reduction in efficiency observed for strongly asymmetric flow distributions is explained by the sharp decrease in turbine efficiency at low relative flows, which is characteristic of Crossflow turbines.

The obtained results provide a theoretical justification for practical operating rules of small hydropower plants with Crossflow turbines and can be applied in the design and operation of control systems for such facilities.

Keywords: crossflow turbine, small hydropower plant (SHPP), parallel operation, flow distribution, efficiency optimization, Lagrange multipliers, operating regimes.

Introduction

Installation of one aggregate at SHPP supplying the grid with generated electricity is the most economically optimal option. Therefore, aggregates equipped with Crossflow turbines have proven themselves in the construction of small hydropower plants on high-mountain rivers due to their stable efficiency over a wide range of flow rate fluctuations and their low starting threshold, which is only 5-6% of the design flow.

The runoff of high-mountain rivers in arid regions is generally characterized by large fluctuations between flood and low-flow periods. Therefore, to rationally cover the range of flow variations, even when Crossflow turbines are installed, it is sometimes necessary to install two or more aggregates.

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When installing more than one aggregate, to ensure maximum plant efficiency, arises the issue of optimal distribution of flow among the turbines.

An issue of flow distribution between aggregates is the category of optimal load/water distribution between hydroaggregates and is solved using constrained optimization methods. However, Crossflow turbines are characterized by a specific form of partial efficiency and control, so the direct application of general recommendations requires adaptation and verification using real characteristics.

Let us consider the case of two aggregates installed at SHPP. Then, the total capacity of the plant, assuming a constant design head, is expressed by the relationship:

$$N(q_1 q_2) = gH[q_1\eta_1(q_1) + q_2\eta_2(q_2)], \quad (1)$$

Where: q_1, q_2-are the flows through the first and second aggregates, respectively, H – is the design head of the plant, $\eta_1(q_1), \eta_2(q_2)$ – are the efficiencies of the first and second aggregates. The flows of the aggregates are constrained by the following relationship:

$$q_1+q_2=Q, \quad (2)$$

where:Q – is the plant flow at the current moment.

To account for the total flow limitation, the Lagrange multiplier method is used. Let us introduce the Lagrange function [1–4]:

$$L\ (q_1,q_2,\lambda)=gH[q_1\eta_1(q_1)+q_2\eta_2(q_2)]+\lambda(Q-q_1-q_2), \quad (3)$$

Where λ - Lagrange multiplier, which has the dimensions of capacity for per unit flow rate.

The necessary condition for an extremum is that the partial derivatives of the Lagrange function with respect to the variables $q_1$and q_2 are equal to zero:

$$\frac{\partial L}{\partial q_1} = gH\left[\eta_1(q_1) + q_1\frac{d\eta_1}{dq_1}\right] - \lambda = 0,$$

$$\frac{\partial L}{\partial q_2} = gH\left[\eta_2(q_2) + q_2\frac{d\eta_2}{dq_2}\right] - \lambda = 0, \quad (4)$$

By eliminating the Lagrange multiplier λ, the key equation for optimal flow distribution is obtained:

$$\eta_1(q_1)+q_1\eta_1'(q_1)=\eta_2(q_2)+q_2\eta_2'(q_2), \quad (5)$$

This relationship is a necessary condition for an interior optimum and is valid for arbitrary efficiency characteristics of the aggregates.

Let us consider a particular, but practically the most important case, when both hydropower aggregates are identical in terms of their energy characteristics. This is the most common design solution, as it ensures interchangeability of spare parts between the aggregates. In this case, relationship (6) can be written in the following form:

$$N(q_1 q_2) = gH[q_{1i}\eta_{1i}(q_{1i}) + (Q - q_{1i})\eta_{2i}(Q - q_{1i})] \rightarrow \max, \quad (6)$$

Moreover, the range of variation of the values q_i is as follows:

$$q_1 \leq q_{1i} \leq Q; \;\; 0 \leq q_{2i} \leq q_2,$$

Considering that the design head of the plant is constant for each i-th value of Q_i, the problem reduces to solving the following relationship:

$$F(q_{1,}q_2) = [q_{1i}\eta_{1i}(q_{1i}) + (Q - q_{1i})\eta_{2i}(Q - q_{1i})] \rightarrow \max \;, \tag{7}$$

The literature presents numerous examples of efficiency curves for Crossflow turbines, which very often do not correspond to each other [5,6]. These discrepancies arise because the authors identify Banki turbines with modern Crossflow turbines equipped with a dual regulation system.

In order to avoid such errors, the efficiency curves used in this study are based on passport data (guaranteed by the manufacturer) from more than 30 different projects, covering a wide range of design flows and heads. The technical characteristics of aggregate components with different parameters were analyzed [7], and as a result of the study it was found that high-quality Crossflow turbines, although differing in nominal efficiency, exhibit similar characteristics in terms of the dynamics of efficiency reduction with changing flow. This makes it possible to unify the efficiency curves $\eta = f(h)$, by introducing a virtual efficiency equal to:

$$\eta_v = \frac{\eta_i}{\eta_n}, \tag{8}$$

Where: η_v, η_n – are the virtual and nominal efficiencies of the aggregate, respectively.

Figure 1 presents the values of the virtual efficiency of anaggregate equipped with a Crossflow turbine and of its individual components.

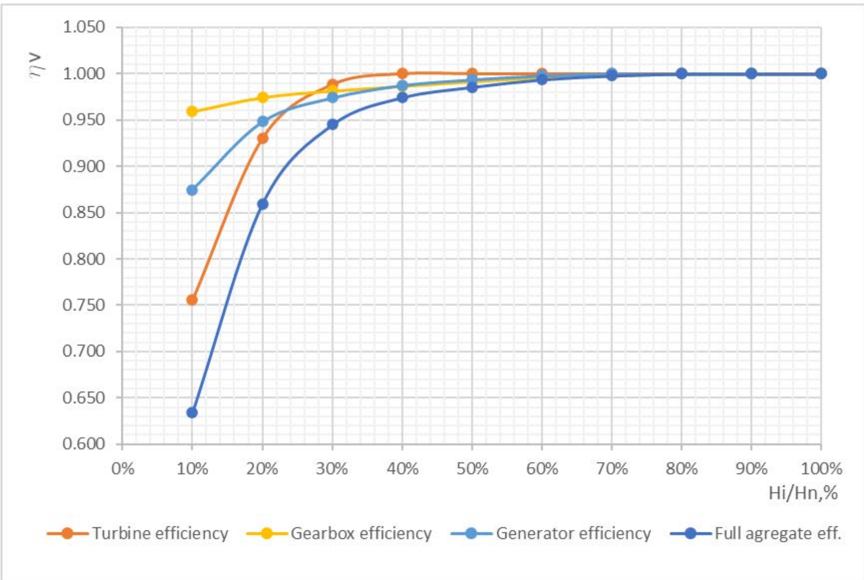

Fig. 1 Virtual efficiency of anaggregate equipped with a Crossflowturbine and its individual components

The design features of the aggregate, including the type of guide vanes and its operating modes, are not explicitly considered in the subsequent analysis, since their influence has already been taken into account in the experimental dependence of efficiency on flow.

The studies have shown that the curves of variation of the virtual efficiency values are well approximated by a sixth-degree polynomial. Table 1 presents the coefficients of the polynomial approximations of the virtual efficiency curves of the aggregate as a whole and of its individual components.

As can be seen from Fig. 1, in contrast to other types of turbines (for example, Francis turbines [5]), the efficiency curve of an aggregate equipped with a Crossflow turbine exhibits a continuously increasing character.

It follows that, when the plant flow fluctuates within the range $0 \leq Q \leq q_1$, the maximum power capacity is achieved exclusively by operating a single unit, and there is no need for dispatching control of the station.

A program for the numerical solution of relationship (7) was developed in the VBA environment. The calculation results obtained for a plant with two aggregates are presented in Fig. 2.

Table 1

Coefficients of the sixth-degree polynomial for anaggregate equipped with a Crossflow turbine and its components.

K of polynomial	Aggregate	Turbine	Gearbox	Generator
A0	0.115059	0.335271	0.921054	0.691226
A1	7.400958	6.055544	0.574933	2.720816
A2	-26.76362	-22.3743	-2.45923	-11.0335
A3	51.79591	43.0003	5.96336	24.56213
A4	-55.66332	-45.4277	-7.79015	-30.4424
A5	31.3024	25.06336	5.136381	19.61107
A6	-7.187377	-5.65245	-1.34637	-5.10938
R^2	**1**	**1**	**0.9998**	**0.9999**

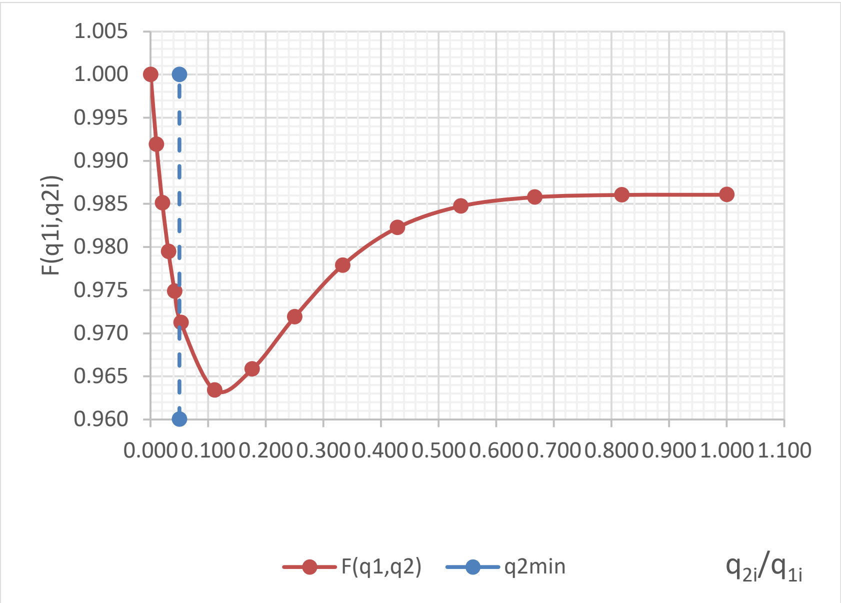

Fig.2 Dependence of F(q1, q2) on the ratio q2i/q1i)

As can be seen from Fig. 2, the target function sharply begins to decrease at small values of the ratio q2i/q1i and reaches its minimum value a$q_{2i}/q_{1i} \simeq 0.125$. Further, the value of $F(q_{1,}q_2)$ begins to increase and reaches its maximum value at $q_{2i}/q_{1i} = 1$.
Taking into account the minimum starting flow of Crossflow turbines, equal to 5%, it can be concluded that in the flow range $q_1 \leq Q_i \leq Q$ is necessary to equalize the turbine flows, which provides the maximum possible capacity of SHPP.

Thus, for 0≤Q≤q1 the optimal operating regime is realized with the operation of a single aggregate, whereas in the rang q1≤Q≤2q1 the optimization problem becomes nontrivial, and the maximum capacity is achieved with an even distribution of flows between the aggregates.
Efficiency curves of the plant with and without dispatching control were constructed by the authors (Fig. 3).

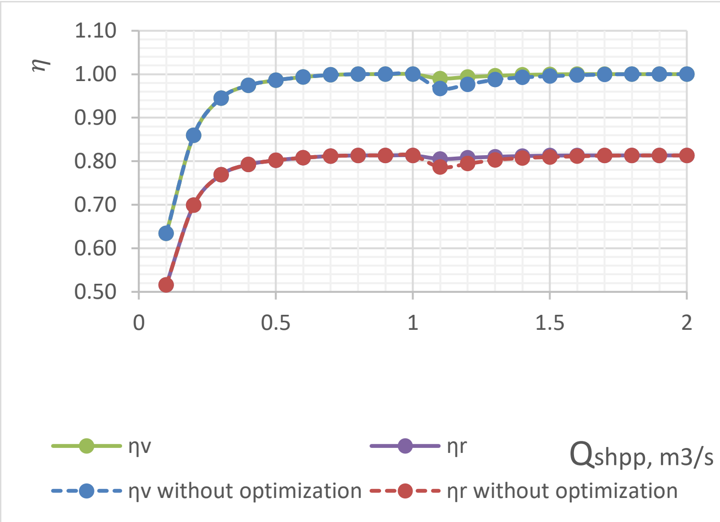

Fig. 3 Efficiency curves of the plant with two aggregates with and without dispatch control

The analysis of the calculation results shows that, due to the stable efficiency of Crossflow turbines, the influence of the absence of dispatch control during parallel operation of aggregates is relatively small compared to other types of turbines.

The calculations have shown that, when two identical aggregates are installed, the absence of dispatch control has the greatest effect in the flow range $q_1 \leq Q_i \leq 1.5Q$. In this range, the maximum efficiency drop reaches 2.3% at a flow of approximately $Q \simeq 1.1q$. Beyond this range, the influence of dispatching control practically does not affect the total capacity of the plant.

Limitations: All solutions presented above are obtained for Crossflow turbines equipped with a two-section guide apparatus and cannot be applied to other design configurations.

1. The developments are carried out for aggregates equipped with a gearbox. In the absence of a gearbox, it is recommended to recalculate the polynomial coefficients and to set the gearbox efficiency value $\eta_{gb} = 1$ in the gearbox efficiency term.
2. The obtained solutions are not applicable to aggregates operating at heads below 40 m, where Crossflow turbines operate in a reaction mode.

Conclusion

1. When the plant flow fluctuates within the nominal flow of a single aggregate, the maximum capacity of a SHPP is achieved by operating one aggregate, and dispatch control is not required.

2. When the total plant flow exceeds the nominal flow of one aggregate, a zone of parallel operation of two aggregates arises, in which the problem of optimal flow distribution becomes essential.
3. Based on the analysis of the energy characteristics of the aggregates and the application of constrained optimization methods, it has been obtained that for identical aggregates the optimal operating regime in the parallel operation zone corresponds to an even distribution of flow between the aggregates.
4. Numerical analysis of the functional proportional to the plant capacity has confirmed the presence of a maximum of the total capacity at equal flows through the aggregates, which is clearly demonstrated by graphical dependencies.
5. The obtained results make it possible to substantiate a practical operating rule for SHPP with Crossflow turbines: when transitioning to the parallel operation of aggregates, it is advisable to ensure an even distribution of water flow in order to achieve the maximum total capacity of the plant.

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«CROSSFLOW» ՏՈՒՐԲԻՆԱՅԻՆ ԱԳՐԵԳԱՏՆԵՐԻ ԶՈՒԳԱՀԵՌ ԱՇԽԱՏԱՆՔԻ ՕՊՏԻՄԱԼԱՑՈՒՄ

Ս.Գ.Գաբայան

Ակադեմիկոս Ի.Վ. Եղիազարովի անվան ջրային հիմնահարցերի և հիդրոտեխնիկայի ինստիտուտ

Հոդվածում ուսումնասիրվում է փոքր հիդրոէլեկտրակայանի Crossflow տուրբիններով համալրված երկու նման հիդրոագրեգատների զուգահեռ աշխատանքի օպտիմալացման խնդիրը՝ ջրի փոփոխվող հոսքի պայմաններում: Հետազոտության նպատակն է որոշել կայանի ընդհանուր հոսքի այնպիսի բաշխում, որը տվյալ հոսքի դեպքում կապահովի առավելագույն հզորություն: Օպտիմալացման խնդիրը ձևակերպվում է Լագրանժի բազմապատկիչների մեթոդի կիրառմամբ, որտեղ նպատակային ֆունկցիան համեմատական է հիդրավլիկական հզորությանը՝ հաստատուն ճնշման պայմաններում:

Ցույց է տրվում, որ կայանի այն հոսքի դեպքում, որը չի գերազանցում մեկ ագրեգատի անվանական հոսքը, առավելագույն հզորությունը ստացվում է մեկ ագրեգատի աշխատանքի պայմաններում, և չի պահանջվում հսկողություն կամ կառավարում: Երկու ագրեգատների համատեղ աշխատանքի տիրույթում առաջանում է ոչ տրիվիալ օպտիմալացման խնդիր: Crossflow տուրբինների արտադրողի տեխնիկական բնութագրերի վրա հիմնված թվային վերլուծությունը ցույց է տալիս, որ տվյալ տիրույթում առավելագույն ընդհանուր հզորությունը ապահովվում է հոսքի հավասար բաշխման դեպքում: Արդյունավետության նվազումը հոսքի խիստ անհավասար բաշխման ժամանակ պայմանավորված է Crossflow տուրբիններին բնորոշ՝ փոքր հարաբերական հոսքերի դեպքում օգտակար գործողության գործակցի կտրուկ անկմամբ:

Ստացված արդյունքները հնարավորություն են տալիս տեսականորեն հիմնավորել փոքր հիդրոէլեկտրակայանների գործնական շահագործման ռեժիմները և կարող են կիրառվել նման կայանների նախագծման և կառավարման գործընթացներում:

Բանալի բառեր. Crossflow տուրբին, փոքր հիդրոէլեկտրակայան, ագրեգատների զուգահեռ աշխատանք, հոսքի բաշխում, հզորության օպտիմալացում, Լագրանժի մեթոդ, շահագործման ռեժիմներ

Оптимизация параллельной работы агрегатов, оснащенных турбинами «CROSSFLOW»

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В работе рассматривается задача оптимизации параллельной работы двух идентичных гидроагрегатов, оснащённых турбинами Кроссфлоу, при переменном расходе воды на МГЭС. Целью исследования является определение рационального распределения суммарного расхода станции между агрегатами, обеспечивающего максимальную вырабатываемую мощность при заданном расходе. Задача оптимизации формулируется с

использованием метода множителей Лагранжа, при этом целевая функция пропорциональна гидравлической мощности при постоянном напоре.
Показано, что при расходе станции, не превышающем номинальный расход одного агрегата, максимальная мощность достигается при работе одного агрегата и необходимость диспетчеризации отсутствует. В диапазоне совместной работы двух агрегатов возникает нетривиальная задача оптимального распределения расхода. Численный анализ, выполненный на основе паспортных характеристик турбин Кроссфлоу, показывает, что в данной зоне максимальная суммарная мощность достигается при равномерном распределении расхода между агрегатами. Снижение эффективности при существенно неравномерном распределении расхода обусловлено резким падением коэффициента полезного действия турбины в области малых относительных расходов.
Полученные результаты позволяют теоретически обосновать практические режимы эксплуатации малых гидроэлектростанций с турбинами Кроссфлоу и могут быть использованы при проектировании и управлении такими объектами.

***Ключевые слова*:** турбина Кроссфлоу, малая гидроэлектростанция, параллельная работа агрегатов, распределение расхода, оптимизация мощности, метод Лагранжа, эксплуатационные режимы.

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