UDC – 627:532.593.3

A FINITE ELEMENT METHOD IN THE STUDY OF NEARSHORE WAVE PROCESSES

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https://doi.org/10.56243/18294898-2022.4-67

Abstract

The paper suggests that the only feasible method for implementing adequate nearshore wave dynamics models under conditions of extreme complexity is numerical methods with computational experiments on powerful computers. When the fundamental laws of continuum mechanics are roughly written for a finite element (FE) with an emphasis on the ensuing numerical solution, the finite element method (FEM) offers great opportunities in this regard.

The FEM has the benefit of allowing for the well-approximation of coastal water areas by a collection of irregular triangles, and their boundaries can typically be curvilinear. The FEM has the advantage that its grid equations typically are independent of the type of grid and its topology, setting it apart from other grid methods.

The generalized solutions to the original problems are divided into grid-like equations for the FEM, which are derived on the basis of integral relations. The fundamental integral laws are thus automatically maintained for grid equations. For the study of the coastal wave regime in a non-stationary three-dimensional formulation, grid equations of the FEM are constructed in this work.

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Key words: water, wave, cxope, finite element, numerical methods.

Intriduction

The nearshore differs significantly from deep water areas and estuaries, which justifies special consideration for them despite the fact that many physical and mathematical models are valid for a wide range of conditions.

The shallow depth of the nearshore - typically 20 to 30 meters - distinguishes it from the open sea's typical depths of 1000–2000 meters. The marine boundary of the continental shelf is typically identified by a sharp rise in the bottom slope, which ranges from 1/500 to 1/10 on average. In comparison to deep water areas, the bottom, which is located at a relatively shallow depth, significantly restricts the movement of water. The near-bottom current, which is insignificant in the open sea, becomes significant here because bottom currents are typically quite strong.

The presence of a coastline restricts water's ability to flow in a direction perpendicular to it, forcing the currents to diverge and orient themselves along the coast. Restrictions on the movement of water towards the coast leads to the occurrence of a level slope, and this in turn causes changes in the dynamics of the nearshore waters. Different regions experience the coast's influence in different ways.

River runoff leads to a decrease in salinity, and hence the density of sea water. For the same values of the heat flux through the sea surface, shallower water areas near the coast experience greater temperature variations than deep water areas. As a result, coastal waters are frequently places with noticeable horizontal gradients in salinity, temperature, and density, which frequently result in changes to the nature of currents.

Coastal waters are also of particular economic and environmental importance. Thus, port facilities are being built in the coastal zone. The coast is frequently used for recreation, including well equipped beaches, swimming zones, and other forms of entertainment. The shelf zone and the marginal seas are where most of the fishing occurs.

Breakwaters, piers, walls, and other necessary structures are built to ensure the safe operation of ports. For the design and operation of these structures, it is essential to understand the heights, periods, and wave approach directions in a particular area. In addition to having a significant impact on coastal structures, waves also have an impact on the movement of beach and bottom material, which can result in erosion in some areas and flooding in others. Unrest is extremely difficult to predict, but forecasting techniques are essential.

Conflict Setting

Calculating the wave regime is the first and one of the most important components of the lithodynamics model (bank abrasion and accumulation, bottom erosion and sedimentation, transport of sediment and suspension) [1,2,6]. In conditions of extreme complexity of adequate models of wave dynamics in the coastal zone [3], numerical methods with computational experiments on powerful computers are typically the only way to implement them.

Building a model while considering the numerical method and algorithm for its implementation therefore seems quite justified. When the fundamental laws of continuum mechanics are roughly written for a finite element (FE) with an emphasis on the ensuing numerical solution, the finite element method (FEM) offers good opportunities in this regard [4].

The FEM belongs to the class of projection methods with unique coordinate functions possessing a final carrier. The prospectivity of this method for issues involving coastal dynamics is conditioned by a number of its significant characteristics and which can give the numerical results of a fresh quality.

When modeling near-shores, the domain of integration is frequently a complex cluster of interconnected domains. In this regard, FEM has the benefit of accurately approximate the coastal water area using a collection of irregular triangles with curvilinear boundaries.

Continuous distributions of the given and desired parameters cover the entire area for continuum objects. Therefore, using approximations, which are also defined across the entire integration domain, is natural. The FEM approximate solutions have this characteristic. For them, the issue with accurate finite-difference approach interpolation of the continuum solution is solved.. The advantage of the FEM, which distinguishes it from other grid methods, is that the grid equations of the FEM, as a rule, do not depend on the type of grid and its topology. This leads to wide possibilities in automating the construction of FEM schemes and introducing grids adaptable to solutions.

The grid equations of the FEM are obtained on the basis of integral relations that determine the generalized solutions of the original problems. Therefore, for grid equations, the basic integral laws are automatically preserved.

Our primary tool for creating numerical algorithms was the FEM. In the section below, a nonstationary three-dimensional formulation of the coastal wave regime is studied. To do this, grid equations of the FEM are constructed. The comparative benefits of the FEM in the investigation of oceanic processes have been considered [5].

Research Results

Let the rectangular Cartesian oocrdinates of a point in a continuous medium, which are obtained when the sea is calm, be its material (Lagrangian) x^1, x^2, x^3 coordinates (initial configuration). Let's divide the medium's continuous region into finite elements so that their motion can be studied (Fig. 1).

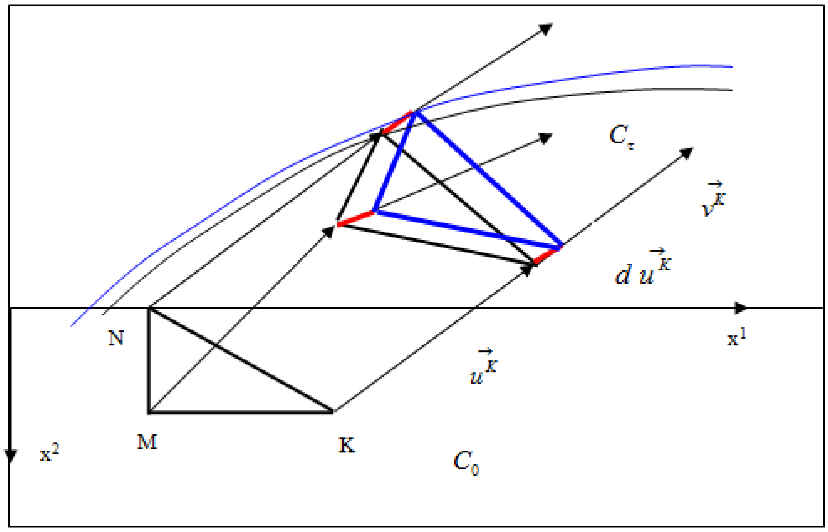

Fig. Scheme of a continuous medium element motion

The general equation of motion for a finite element (FE) in a continuous medium has the following structure [4].

$$m_{MN}\frac{dv_i^M}{d\tau}+\int\limits_{V_{o(e)}} t^{mj}\psi_{N,m}\left(\delta_{ji}+\psi_{M,j}u_i^M\right)dV_0=p_{Ni}, \tag{1}$$

where $u_i^M(\tau), v_i^M(\tau)$ are components of the displacement and velocity vectors of node M; $\psi_N(x^1,x^2,x^3)$ are local interpolation (basic) functions.

Substituting $t^{mj}=-\psi_L p^L G^{mj}+\mu G^{mk}G^{Ji}\left(\psi_{P,k}v_i^P+\psi_{P,i}v_k^P\right)$ into Eq.(1), we have the relation between the components of the stress tensor and strain rates for an isotropic incompressible viscous fluid [5] in a finite element form: (here $p^L(\tau)$ is the pressure at the node, L; G^{mj} are contravariant components of the metric metric tensor, in the current configuration C_τ), for the given case equation of the FE motion is obtained

$$m_{MN}\frac{dv_i^M}{d\tau}+a_{NiP}^k v_k^P-b_{NiL}p^L=p_{Ni}, \tag{2}$$

where the mass matrix for FE is $m_{MN}=\int\limits_{V_{o(e)}}\rho_0\psi_N\psi_M dV_0$,

coefficients at nodal velocities

$$a_{NiP}^k=\int\limits_{V_{0(e)}}\mu(G^{ms}G^{jk}+G^{mk}G^{js})(\delta_{ji}+\psi_{M,j}u_i^M)\psi_{P,s}\psi_{N,m}dV_0,$$

coefficients at nodal pressures

$$b_{NiL}=\int\limits_{V_{0(e)}}G^{mJ}\left(\delta_{ji}+\psi_{M,j}u_i^M\right)\psi_L\psi_{N,m}dV_0,$$

components of the generalized force at the node N [4]

$$P_{Ni}=\int\limits_{V_{0(e)}}\rho_0F_i\psi_N dV_0+\int\limits_{A_{0(e)}}S^J\left(\delta_{ji}+\psi_{M,j}u_i^M\right)\psi_N dA_0, \tag{3}$$

where F_i and S^j are components of intensities of external volumetric and surface forces acting on a finite element and behaving on per unit, respectively, of volume and area in the initial configuration C_0.

To obtain the equilibrium equations for the entire ensemble of $V_{0(e)}$ elements, we will link the elements into a common region V_0 [4]. The interlinking specifies the relation of incidence defined by the function $\overset{(e)}{\Omega}{}_\Delta^N$, which takes the value 1, if the $V_{0(e)}$ element's local node N is the same as the global node Δ of the linked region V_0, and the value 0 otherwise. If the local coordinate lines of the elements coincide with the global coordinate lines of the entire region V_0, then the local values of the components are linked by relations, for example, for velocities $v_{i(e)}^N=\overset{(e)}{\Omega}{}_\Delta^N V_i^\Delta$ (here, the summation is made by Δ - over all global nodes).

The global values of the components of the generalized forces in the node Δ of the linked model is obtained by summing up the efforts of all local nodes coinciding with the node Δ [4]:

$$P_{\Delta i} = \sum_{(e)} \overset{(e)}{\Omega}{}^{N}_{\Delta} \, p_{Ni} \tag{4}$$

Substituting Eq.(2) into Eq.(4), we get the global equations of motion for the entire ensemble of finite elements

$$\sum_{(e)} \overset{(e)}{\Omega}{}^{N}_{\Delta} \left(m_{NM} \frac{dv_i^M}{d\tau} + a_{NiP}^{k} v_k^P - b_{NiL} p^L \right) = P_{\Delta i} . \tag{5}$$

The finite element analogue of the incompressibility condition at each point of a continuous medium (local condition) $\frac{d}{d\tau}\left(\det G_{ij}\right)$ can be written as

$$\sum_{(e)} \overset{(e)}{\Omega}{}^{N}_{\Delta} \int_{V_{0(e)}} \psi_N^{(e)} \frac{d}{d\tau}\left(\det G_{ij}\right) dV_0 = 0 \tag{6}$$

Note that Eq.(6) is a linear equation with respect to nodal velocity components. For example, for the flat case we have

$$\frac{d}{d\tau}\left(\det G_{\alpha\beta}\right) = G_{11}\frac{dG_{22}}{d\tau} + G_{22}\frac{dG_{11}}{d\tau} - 2G_{12}\frac{dG_{12}}{d\tau} ,$$

$$G_{\alpha\beta} = \delta_{\alpha\beta} + \psi_{N,\alpha} u_\beta^N + \psi_{M,\beta} u_\alpha^M + \psi_{N,\alpha}\psi_{M,\beta} u_\gamma^N u_\gamma^M ,$$

$$\frac{dG_{\alpha\beta}}{d\tau} = \psi_{N,\alpha} v_\beta^N + \psi_{M,\beta} v_\alpha^M + \psi_{N,\alpha}\psi_{M,\beta} u_\gamma^M v_\gamma^N + \psi_{N,\alpha}\psi_{M,\beta} u_\gamma^N v_\gamma^M .$$

It should be noted that the time term was approximated using the approximator formulas.

As a result, the calculation could be simplified and the Cauchy problem could be considered in the future as opposed to a more general evolutionary problem with an averaging operator and a time term. The approximation properties remain unchanged in this instance, as is well known.

Conclusions

As a result, the linear global q. (5) and (6) of motion for the entire ensemble of finite elements were obtained to study the wave motions of the sea in the nearshore. The Crank-Nicolson schemes [4], which offer the second order of approximation in time, can be used to construct time approximations linked with the solution of Eqs. (5) and (6). The upper block relaxation method successfully solves the resulting algebraic equations with the proper boundary conditions and initial data.

The validity of Eqs.(5) and (6) holds true for boundary nodes that are situated on the bottom and the free surface. However, unlike internal nodes, where the surface forces are entirely balanced when the elements are connected and contribute zero to $P_{\Delta i}$, the surface forces at these nodes are calculated taking into account the influence of the atmosphere and bottom. For instance, surface forces are calculated by neglecting wind shear stresses and atmosphere pressure p0 for surface forces in (3)

$$S^j = -p_0 \sqrt{G} G^{jk} n_k$$

where n_k is a unit vector component of the normal to the dA_0 area of free surface in the initial geometric pattern.

Different methods can be used in the model to describe the interaction with the seabed, and these methods will be reflected in the recording of the corresponding relationships for boundary nodes. For instance, nodal displacements and velocities at the boundary are equal to zero when using the kinematic condition of particles adhering to a solid bottom. In this situation, in the absence of relative movement at the bottom, bottom stresses can be estimated using Eqs.(5) and (6).

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ԾՈՎԻ ԱՓԱՄԵՐՁ ԳՈՏՈՒՄ ԱԼԻՔԱՅԻՆ ՊՐՈՑԵՍՆԵՐԻ ՀԵՏԱԶՈՏՈՒԹՅԱՆ ԽՆԴԻՐՆԵՐԻ ԴՐՎԱԾՔԸ ՎԵՐՋԱՎՈՐ ՏԱՐՐԵՐԻ ՄԻՋՈՑՈՎ

Սաղինաձե Ի.Ս.[1], **Գագոշիձե Շ.Ն.**[2], **Կոդուա Մ.Ա.**[2]
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Աշխատանքում առաջարկվում է, որ առափնյա գոտում ալիքների դինամիկայի մոդելների ծայրահեղ բարդության պայմաններում դրանց իրականացման միակ հնարավոր միջոցը հզոր համակարգիչների վրա հաշվողական մեթոդներն են: Այս առումով վերջավոր տարրերի մեթոդը (FEM) մեծ հնարավորություններ է տալիս այն դեպքում, երբ մեխանիկայի հիմնական օրենքները գրվում են վերջավոր տարրի (FE) համար՝ նպատակաուղղված հետագա թվային լուծման վրա: Վերջավոր տարրերի մեթոդն ունի այն առավելությունը, որ թույլ է տալիս ափամերձ ջրային տարածքները մոտարկել անկանոն եռանկյունների միջոցով, երբ դրանց սահմանները, ընդհանուր առմամբ, կարող են լինել կորագիծ: Վերջավոր տարրերի մեթոդով կիրառվող հավասարումները, որպես կանոն, կախված չեն ցանցի տեսակից և դրա տոպոլոգիայից:

Բանալի բառեր. համալսարան, պատենտ, հետազոտություն, ռազմավարություն:

КОНЕЧНОЭЛЕМЕНТНЫЕ ПОСТАНОВКИ ЗАДАЧ ДЛЯ ИССЛЕДОВАНИЯ ВОЛНОВЫХ ПРОЦЕССОВ В ПРИБРЕЖНОЙ ЗОНЕ МОРЯ

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В работе предлагается, что в условиях чрезвычайной сложности адекватных моделей динамики волн в прибрежной зоне единственным возможным способом их реализации, являются численные методы с проведением вычислительных экспериментов на мощных компьютерах. В этом плане большие возможности предоставляет метод конечных элементов (МКЭ), когда основные законы механики сплошной среды приближенно записываются для конечного элемента (КЭ) с ориентацией на последующее численное решение. МКЭ обладает тем достоинством, что позволяет хорошо аппроксимировать прибрежные акватории набором нерегулярных треугольников, причем их границы могут быть, вообще говоря, криволинейными. Достоинством МКЭ, отличающим его от других сеточных методов, является то, что сеточные уравнения МКЭ, как правило, не зависят от вида сетки и ее топологии.

Сеточные уравнения МКЭ получаются на основе интегральных соотношений, определяющих обобщенные решения исходных задач. Поэтому для сеточных уравнений автоматически сохраняются основные интегральные законы. данном работе проводится построение сеточных уравнений МКЭ для исследования прибрежного волнового режима в нестационарной трехмерной постановке.

***Ключевые слова**:* волна, берег, конечный элемент, численные методы.

Submitted on 12.09.2022.
Sent for review on 19.09.2022.
Guaranteed for printing on 18.12.2022.