UDC - 627.521.1

ON THE STABILITY OF WATER MOVEMENT IN FREE-FLOW CONDUITS OF CIRCULAR CROSS SECTION

Shalva N. Gagoshidze
Georgian Thecnical University
77 Kostava street, Tbilisi, 0171
sh.gagoshidze@gmail.com
ORCID iD: 0000-0002-3223-7817
Republic of Georgia

Manoni A. Kodua
Georgian Thecnical University
77 Kostava street, Tbilisi, 0171
m.kodua@gtu.ge
ORCID iD: 0000-0002-3186-9062
Republic of Georgia

https://doi.org/10.56243/18294898-2022.3-51

Abstract

The article considers the stability of fluid movement in free-flow conduits of circular cross section. Using the method of wave perturbations, for the first time, it is mathematically substantiated why the flow in tunnels or pipelines of circular cross section occurs intermittently when they are almost completely filled. The obtained theoretical results are consistent with existing experimental and field observations, according to which in free-flow conduits of circular cross section, when they are filled by more than 92–93%, water always moves intermittently, with bursts to the ceiling, i.e., it is unstable.

Asymptotic equations are also displayed for describing wave movements in half or to a small degree of round cross -sectional channels, a qualitative analysis of which indicates the stability of the surface waves that arose in them.

Key words: non-pressure motion, circular section, wave disturbances, sustainability.

Introduction

The protection of the soil from a variety of harmful factors, such as washing, wind drift.

The capacity of tunnels and pipelines operating in a non-pressure mode depends significantly on the stability of fluid movement in them. In general, the study of stability issues is not limited to consideration of simple schemes. This is a very complex problem, which lies, first of all, in the formulation of mathematical criteria for the stability of the motion of solids, particles of liquids, gases or molecules. Unlike hydraulic methods, the study of the stability of flows, which are developed in the works of Voynich-syanozhnsky [1] and Kartvelishvili [2] and in which the influence of the hydraulic index of the channel on the stability of water flows is studied, the study of the stability of flows in free-flow tunnels by more "clean" - hydrodynamic methods is associated with the use of a very complex mathematical apparatus. This applies to the study of wave disturbances even in seemingly simple conduits, such as a tunnel or a conduit with a semicircular cross section (Lamb [3]).

In hydrodynamics, in addition to the criteria of absolute stability and instability of motion, developed by Lyapunov at the end of the 19th century for material particles, the Kelvin-Helmholtz stability criterion [3] is widely used, which, within the framework of a plane problem, determines the possibility of the existence of internal periodic waves of constant length in time and depends on the oscillation frequency of these waves. If the frequency, depending on the difference in flow velocities, takes a complex (imaginary) value, then the surface of internal waves increases infinitely in time and the movement becomes unstable. It should be noted that the Kelvin instability is adequate to the absolute Lyapunov instability, which cannot be said about stability, since, according to Lyapunov, stability means maintenance, i.e. return to the mirror (unperturbed) interface of these flows after removal of perturbations from this surface..

The study of the stability of flows in channels is directly related to the study of the propagation of surface along-shore waves, the exact solutions of which, as noted above, are limited only to cases of triangular channels with slope angles of sides to the horizon of 45° and 60°. As for non-pressure channels of a circular cross section, as one of the founders of the theory of wave motion of liquids, the great American scientist George Lamb, noted, «the wave motion in them has not been studied even for such a seemingly simple cross section as a semicircular cross section».

Conflict Setting

The present work is devoted to filling the existing gap in this direction, in which three, practically very important cases of propagation of surface waves in channels of a circular cross section are considered by asymptotic methods. In particular, the case of the presence of:

1. Channel of circular cross section almost completely filled with water;
2. A channel of circular cross section with a very shallow depth of water flow;
3. Channel of circular cross-section is half filled with water flow.

Research Results

1.System of basic equations. As noted above, the wave motion of a fluid in most cases makes it possible to ignore the viscous forces. This assumption greatly simplifies the equations of the dynamics and at the same time allows us to attribute a wave movement to the class without vortex potential movements. This means that the velocity field at the passing point occupied by the liquid can be determined by one vector equality

$$\vec{V} = grad\phi, \tag{1}$$

where $\vec{V}$ is the velocity vector of water particles, ϕ is the potential of the velocity field, which in our case, in addition to the wave motion of the liquid, is due to the motion of the liquid at a constant speed U_0.

In the general case, if we choose the Cartesian coordinate system, in which the axis z is directed vertically upward from the center of the conduit of circular cross section, and the axes x and y are aligned with the equatorial plane so that the direction of the axis x coincides with the direction of flow (Fig. 1), then equality (1) for the velocity components u, v, w will be written in the form

$$u = U_0 + \frac{\partial \varphi}{\partial x}; \qquad v = \frac{\partial \varphi}{\partial y}; \qquad w = \frac{\partial \varphi}{\partial z} \quad , \tag{2}$$

where φ is the desired potential of the speed of wave motion, with respect to which the system of linear equations of wave motion takes the following form:

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0; \tag{3}$$

$$\frac{\partial^2 \varphi}{\partial t^2} + U_0^2 \frac{\partial^2 \varphi}{\partial x^2} + 2U_0 \frac{\partial^2 \varphi}{\partial x \partial t} = -g \frac{\partial^2 \varphi}{\partial z^2}, \text{ at } z = \pm h; \tag{4}$$

$$\frac{\partial \varphi}{\partial n} = 0, \text{ at } z = R_0(x, y), \tag{5}$$

Equation (3) is the Laplace equation, which is valid at an arbitrary point occupied by the fluid; Equation (4) (where t time) is a dynamic boundary condition on the wave surface of a moving fluid, i.e. on the surface , where is the coordinate of the wave surface measured from the mark of the undisturbed level of the water flow in the tunnel and which is a negligible value compared to the depth of the center of the tunnel circle. The "+" sign is accepted when the tunnel is more than half filled with water flow, and the "-" sign is otherwise; Equality (5) is the condition of non-flow of the inner cylindrical surface of the tunnel.

For further transformations, it is more convenient to write the above equations in a cylindrical coordinate system x, r and θ the relationship of which with Cartesian coordinates is expressed by the following equalities:

$$x = x; \quad y = r\cos\theta; \quad z = r\sin\theta. \tag{6}$$

Here it is assumed that the axis again coincides with the longitudinal axis of the circular duct (Fig. 1).

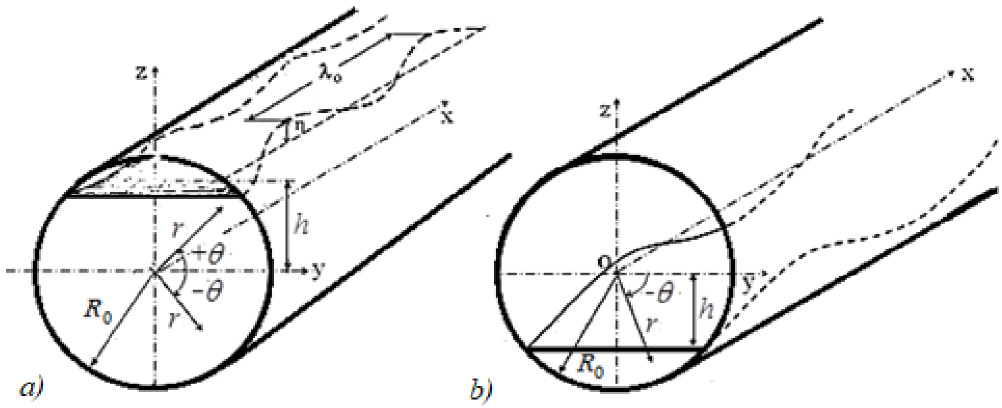

Fig.1 The calculation schemes of the wave movement of the flow in the non -pressure water conduit of the round cross section.

a) an almost completely filled waterflower;
b) a water water with a small filling compared to the radius

The radius vector r originates at the center of the circle, and the polar angle θ is measured from the horizontal diameter in the opposite clockwise direction. In such a coordinate system, the equations of the unperturbed free surface of the flow, depending on whether this surface is above or below the horizontal axis x, respectively, is written in the form $r\sin\theta = +h$ and $r\sin\theta = -h$, and the cylindrical surface of the conduit is expressed by the equality $r = R_0$, where R_0 is the inner radius of the conduit.

As a result of standard transformations, the system of equations (3) ÷ (5) in polar coordinates takes the form:

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2\varphi}{\partial\theta^2} = 0; \tag{7}$$

$$\frac{\partial^2\varphi}{\partial t^2} + U_0^2\frac{\partial^2\varphi}{\partial x^2} + 2U_0\cdot\frac{\partial^2\varphi}{\partial x\partial t} = -g\sin\theta\frac{\partial\varphi}{\partial r} - g\frac{\cos\theta}{r}\cdot\frac{\partial\varphi}{\partial\theta}, \text{ at } r\sin\theta = \pm h; \tag{8}$$

$$\frac{\partial\varphi}{\partial r} = 0, \text{ at } \quad r = R_0 \tag{9}$$

The solution of the boundary value problem (7)÷(9) is associated with large, yet insurmountable mathematical difficulties. In the case of water conduits of circular cross section, these difficulties are further aggravated by the fact that it is impossible to choose such a coordinate system in which the cylindrical surface of the conduit and the free surface of the water flow are simultaneously described by linear relations. In particular, if in a cylindrical coordinate system the round inner surface of the conduit is described by a linear formula, then for the horizontal surface of the flow in the conduit we are forced to apply a nonlinear dependence. Other types of transformations cause non-linear changes in the system of basic equations and create new difficulties. For all these reasons, we are forced to confine ourselves to the consideration of the limiting (asymptotic) fillings of a circular water conduit listed above. But before that, let's make general transformations based on the representation of the velocity potential of wave disturbances as a periodic complex function in time t and along the longitudinal coordinate x :

$$\varphi = \psi(r,\theta)\exp[i(\sigma t - kx)], \tag{10}$$

where i is the imaginary unit; $\sigma = 2\pi / \tau$ - frequency of wave oscillations; τ - period; $k = 2\pi / \lambda$ - wave number; λ - wavelength (distance between two adjacent points of the wave surface that are in the same phase).

Taking into account the notation (10), the system of basic equations (11)÷(13) takes a simpler form

$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} - k^2\psi = 0, \tag{11}$$

$$(\sigma - kU_0)^2\psi = g\sin\theta\frac{\partial\psi}{\partial r} + \frac{g\cos\theta}{r}\frac{\partial\psi}{\partial\theta}, \text{ on the surfaces } \quad r\sin\theta = \pm h; \tag{12}$$

$$\frac{\partial\psi}{\partial r} = 0, \quad \text{at} \quad r = R_0 \ . \tag{13}$$

The system of equations (11) ÷ (13) still retains its generality, since it can be used as the basis for studying the movement of waves on the free surface of a free flow in water conduits of a circular cross section and arbitrary filling

2. Stability of water flow in an almost filled non-pressure round cylindrical channel.

If a circular water conduit is filled almost completely, we can assume that the change in the polar angle within the narrow free surface of the flow is insignificant, and its sine and cosine take on the values $\sin\theta \approx 1$ and $\cos\theta \approx 0$. In this case, taking into account condition (10), we can also assume that the function ψ does not change in θ not only on the free surface, but also at any internal point of the fluid and write the system of equations (11) ÷ (13) in the following simplified form:

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial \psi}{\partial r} - k^2\psi = 0; \tag{14}$$

$$(\sigma - kU_0)^2\psi = g\sin\theta\frac{\partial \psi}{\partial r}, \quad \text{at } r = h; \tag{15}$$

$$\frac{\partial \psi}{\partial r} = 0, \quad \text{at } r = R_0 \ . \tag{16}$$

As you can see, due to the small width of the liquid surface, the simplified boundary condition (15), in contrast to (12), is satisfied not on the surface $r\sin\theta = h$, but at a point $r = h$ on the vertical axis of symmetry.

The boundary value problem (14) ÷ (16) is subject to exact solution. In particular, the solution of equation (14) (the Bessel equation) is usually written in modified zero-order Bessel functions (Watson)

$$\psi = C_1 I_0(kr) + C_2 K_0(kr)\ , \tag{17}$$

where C_1 and C_2 are integration constants; The functions $I_0(kr)$ and $K_0(kr)$ ($K_0(kr)$ also called the McDonalds function [6,7, 8]) belong to the class of special functions and are not expressed in terms of elementary functions, which makes them somewhat inconvenient for engineering use, despite the fact that these functions are presented in tabular and graphical forms (Watson [5], Jahnke- Emde-Lesh [6]). Therefore, whenever possible, instead of special functions, they often resort to using their asymptotic representations, which are usually expressed in terms of elementary functions and which correspond to large values of their argument (in our case kr).

These asymptotic formulas have the following form (Matthews-Walker [9]):

$$I_0(kr) = \frac{e^{kr}}{\sqrt{2\pi kr}}\left[1 + O\left(\frac{1}{kr}\right)\right]\ ; \tag{18}$$

$$K_0(kr) = \sqrt{\frac{\pi}{2kr}}e^{-kr}\left[1 + O\left(\frac{1}{kr}\right)\right], \tag{19}$$

where $O\left(\frac{1}{kr}\right)$ denotes an infinite sum by an order of magnitude of small values.

Taking into account expressions (18) and (19), the general asymptotic solution (17) is written as follows:

$$\psi = C_1 \frac{e^{kr}}{\sqrt{2\pi kr}} + C_2 \sqrt{\frac{\pi}{2kr}} e^{-kr}. \tag{20}$$

With the help of the boundary condition (16), which is satisfied on the inner cylindrical surface of a sufficiently large radius, the constants and can be reduced to a single constant . In particular, if we use the rule of differentiation of asymptotic dependences (Stoker [4]), then from the boundary condition (16) we obtain:

$$C_1 \frac{1}{\sqrt{2\pi k}} \cdot \frac{e^{kR_0}}{\sqrt{R_0}} = C_2 \sqrt{\frac{\pi}{2k}} \cdot \frac{1}{\sqrt{R_0}} e^{-kR_0} = \frac{C}{2} \ , \tag{21}$$

from which the constants are easily determined and, consequently, the real part of the desired function takes on the following final output:

$$\psi = C \frac{\sqrt{R_0}}{\sqrt{r}} \cosh k(R_0 - r), \tag{22}$$

Let us now substitute (22) in the boundary condition (15), which is satisfied on the free surface of the liquid. Following its asymptotic derivative, we get:

$$(\sigma - kU_0)^2 \cosh k(R_0 - h) = -g \sinh k(R_0 - h), \tag{23}$$

whose solution with respect to frequency leads to the following dispersion relation:

$$\sigma = kU_0 \pm \sqrt{-gk \tanh k(R_0 - h)} \ . \tag{24}$$

It is the analysis of this dispersion relation that gives us the opportunity to judge the Helmholtz stability of wave motion in a water conduit with a circular cross section filled almost completely. In particular, according to (24), since h is always less than R_0 , and $k > 0$, the value of the expression under the root is negative and, therefore, (24) is a complex number:

$$\sigma = kU_0 \pm i\sqrt{gk \tanh k(R_0 - h)}. \tag{25}$$

If we substitute this frequency value in expression (10), we will see that one of the roots of formula (25), (namely, the root with a negative sign), leads to an exponential growth in time of the potential of wave disturbances

$$\varphi \sim e^{mt}, \quad \text{where } m = \sqrt{gk \tanh k(R_0 - h))} \ , \tag{26}$$

which indicates the instability of the wave motion in an almost filled round cylindrical conduit.

Thus, in the study, an important result was obtained:

If a circular conduit (tunnel or pipeline) is almost completely filled with water, then any disturbance on the surface of this liquid will inevitably increase and lead to a splash of liquid on the ceiling of the conduit.

It is the manifestation of such instability that can explain the decrease in the throughput of round water conduits when they are almost completely filled. In this connection, let us recall the process of pouring water from a bottle, which, as you know, if the bottle is sufficiently inclined, is always accompanied by vibration and a pulsating flow of water.

The results of experimental studies of circular water conduits really confirm the fact that when the water conduit is almost completely filled, it becomes more difficult to measure the flow of water into it. In particular, one of the well-known representatives of the Georgian researcher Chanishvili [10], as a result of his meticulous experiments carried out in 1947, came to the conclusion that “sufficiently reliable experiments are possible only up to filling $0{,}93 \div 0{,}95D$ (D - diameter). With an increase in filling from 0.93÷0.95% to 100%, the experiments became impossible; Neither ventilation holes nor viewing windows helped, and therefore the points corresponding to this regime were plotted only by recalculating the experimental points corresponding to the pressure regime". In addition, it is most remarkable for us that Chanishvili came to the conclusion that “despite the change in slopes (that is, the change in flow rates - the authors), the maximum throughput in the experimental pipeline was observed when the pipeline was filled to 0.92÷0.93% ". At the same time, "the experiments failed to determine the effect of ventilation (i.e., the presence of an air layer - the authors) on the flow resistance in general."

As you can see, our theoretical studies are fully consistent with the results of Chanishvili's experiments. This correspondence is also manifested in the fact that, according to dependence (23), the instability does not depend on the flow velocity; It depends only on the filling of the conduit (i.e., on the thickness $(R_0 - h)$ of the air layer between the free flow surface and the ceiling of the vessel) and on the perturbation wavelength $(\lambda = 2\pi / k)$. As the values of these quantities increase, the coefficient of flow instability decreases.

This feature of the pressureless movement of water in almost filled water conduits of a circular cross section was completely rejected by some well-known experts in hydraulics (for example, Bulow [11]), believing that with an increase in the liquid level in the conduit, the water flow increases monotonically. Other hydraulics (including Chanishvili) believed (and is still accepted in hydraulics courses) that, in accordance with the Shezy law, in circular water conduits, reaching a certain filling (in particular, 95% according to Luger [12]), the water flow begins to decrease, which can hardly be imagined with increasing flow depth.

As a result of our study, when a water conduit with a circular cross section is 90% or more filled with water, the use of the Chezy formula is not allowed, since with such a filling, as shown above, it is in principle impossible to implement a stable stationary mode of water flow in a water conduit with a circular cross section, which corresponds to Chezy formula.

As it follows from [13], the unstable state of the flow persists even after the pipeline is completely filled and the flow passes into the turbulent pressure regime.

3. Stability of water flow in an almost empty channel round-cylindrical shape. In this particular boundary case (Fig. 1.*b*)) the polar angle θ takes values close to 90°, so that the following approximate equalities $\sin\theta \approx -1$ and $\cos\theta \approx 0$ are fulfilled.

In addition, when the water horizon is located below the center of the conduit, in condition (10), before h , the sign "-" must be taken. Taking into account all this, the system of equations (9)÷(11) will take the form:

$$\frac{d^2\psi}{dr^2}+\frac{1}{r}\cdot\frac{d\psi}{dr}-k^2\psi=0\,; \tag{27}$$

$$(\sigma-kU_0)^2\psi=-g\cdot\frac{d\psi}{dr},\ \text{at}\quad r=h\ ; \tag{28}$$

$$\frac{d\psi}{dr}=0,\ \text{at}\ \ r=R_0\,. \tag{29}$$

Here, as well as in equation (14), the second order derivative with respect to the polar angle is neglected as a small value. This neglect is also due to the fact that the boundary conditions (13) and (28), which are satisfied on the free surface of the liquid, do not contain the derivative with respect to the polar angle θ. Thus, the difference between the systems of equations (27)÷(29) and (14)÷ (16) is expressed only in the "-" sign in the boundary condition (25), which is in front of the term $\frac{d\psi}{dr}$.

It is this sign that determines the fundamental difference that the solution of the system of equations (27) ÷ (29) gives in comparison with the solution of the system (14) ÷ (16). We will not go into the details of solving system (27) ÷ (29), since this procedure actually repeats the solution procedure described in the previous paragraph. We present only the final results of the asymptotic solution of system (27) ÷ (29). In particular, as in the previous case, the velocity field potential in complex form will be expressed by the following asymptotic dependence:

$$\varphi=\psi\cdot e^{i(\sigma t-kx)}=c\sqrt{\frac{R_0}{r}}\cosh k(R_0-r)e^{i(\sigma t-kx)}, \tag{30}$$

or, if we confine ourselves to the real part of the solution (30), - the dependence

$$\varphi=c\sqrt{\frac{R_0}{r}}\cosh k(R_0-r)\cos(\sigma t-kx), \tag{31}$$

in which, in contrast to dependence (22), the frequency of wave oscillations is a real number and is calculated by the formula

$$\sigma=kU_0\pm\sqrt{gk\tanh k(R_0-h)}, \tag{32}$$

according to which **the wave motion in a water conduit with a circular cross section, at a moderate speed and with a filling that is small compared to the radius, is stable.**

4. On the stability of the water flow in a half-filled round cylindrical channel. When a water conduit of circular cross section is half filled, it can be assumed that the polar angle θ

of oscillation of the free surface is very small and, therefore, the following approximate equalities $\sin\theta \approx \theta$ and $\cos\theta \approx \pm 1$ are fulfilled (the sign "-" corresponds to the consideration of the semicircle left from the symmetry axis of the channel. We will restrict ourselves to the consideration of the right semicircle, with the sign "+ ", which is identical to the consideration of the left semicircle).

Then, taking into account the approximate equalities $\sin\theta \approx \theta$ and $\cos\theta \approx \pm 1$, the system of basic equations (9)÷(11) takes the form:

$$\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial\varphi}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2\varphi}{\partial\theta^2} - k^2\psi = 0 \; ; \qquad (33)$$

$$(\sigma - kU_0)^2\psi = \frac{g}{r}\cdot\frac{\partial\psi}{\partial\theta}, \qquad \text{at} \quad \theta = 0; \qquad (34)$$

$$\frac{\partial\psi}{\partial r} = 0, \qquad \text{at} \quad r = R_0 \; . \qquad (35)$$

As we can see, the system of equations (33)÷(35) differs significantly from the systems of equations (14)÷(16) and (27)÷(29) considered above. This difference is primarily manifested in the condition on the free surface (34), according to which the oscillation frequency σ depends on the derivative of the potential with respect to the polar angle θ. Boundary conditions (34)÷(35) defined by two different arguments indicate that in the case under consideration the problem can have several solutions.

Even an approximate solution of the system of equations (33)÷(35) is associated with great mathematical difficulties. We will consider here only the possibility of propagation in a semicircular water conduit of such waves, symmetric about the axis, whose velocity potential does not depend on the polar angle θ. In this case, as follows from condition (34), the frequency of wave oscillations should be equal to the product of the wave number and the constant flow velocity $\sigma = kU_0$, which, in this case, indicates the absence of longitudinal waves, not only in the case when the flow velocity $U_0 = 0$, but also in the case , when $U_0 \neq 0$.

Indeed, if we use the equality $\sigma = kU_0$ and substitute it in the substitute it in the expression $\eta = a_0 \sin(\sigma t - kx)$ for the wave surface, we get that $\eta = a_0 \sin k(U_0 t - x)$, according to which, since in our case the velocity $U_0 = x/t$, the coordinate of the wave surface η turns out to be a constant (namely, zero) value.

Conclusion

Mathematically non-rigorous statement, allows us only to assume that the uniform flow of water in half-filled channels of circular cross section is always stable and the flow retains its original mirror surface.

References

1. Voinich-Syanozhentsky T.G. and Togonidze N.V. Transformation of surface waves into currents under conditions of depth change (1969) //Tr. ZakNIGMI, vol. 32(38), Gidrometeoizdat, L: p. 132-149. (in Russian)

2. Kartvelishvili N.A. Unsettled open streams (1968) //Leningrad: Gidrometeozdat.- 126 p. (in Russian)
3. Lamb G. Hydrodynamics (1947) //Gostekhizdat, M: 928 p. (in Russian)
4. Stoker, J. J, Water Waves The Mathematical Theory With Applications (1957) //Institute of Mathematical Sciences New York University, New York .- 609 p.
5. Watson G.N. Besel functions (1949) //M: IL.- 276 p.
6. Yanke E., Emde F., Lesh F. Special functions, formulas, graphs, tables (1968) //Translation from German. M: Nauka.- 344 p.
7. Nikiforov A.F., Uvarov V.V. Special functions of mathematical physics (1984) //M: Nauka.- 344 p.
8. Nikiforov A.F., Sidorchuk V.M., Uvarov V.V. Fundamentals of the theory of special functions (1974) //M. Science.- 304 p.
9. Mathews, J. and Walker, R.W.: Mathematical Methods of Physics, Addison-Wesley, Marlo Park, second edition, 1970.
10. Chanishvili A.G. Non-pressure uniform movement of liquids in pipelines (1947) //Izv. TNIGEI, No. 1, Tbilisi, 1947, p. 69-85. (in Russian)
11. Bulow FR. V. Die Leistungfahrigkeit von Flug BachWerkgraben, Kanal und Rohrquerschnitten. Gesundsheits Ingenier, 1927. v. 50 h.262.
12. Evreinov V.N. Hydraulics. Ed. Water transport, M., L. 1939, 632 p.
13. Shevelev M.M., Burinskaya T.M. Nonlinear dynamics of the Kelvin–Helmholtz instability in a plasma flow of finite width. January 2013. Plasma Physics 39(6):546-555

References

1. Войнич-Сяноженцкий Т.Г., Тогонидзе Н.В., Трансформация поверхностных волн на течения в условиях изменения глубин (1969) //Тр. ЗакНИГМИ, вып. 32(38), Гидрометеоиздат, Л: с. 132-149.
2. Картвелишвили Н.А. Неустановившиеся открытые потоки (1968) //Л: Гидрометеоздат.- 126 с.
3. Ламб Г. Гидродинамика (1947) //Гостехиздат, М: 928 с.
4. Stoker, J. J, Water Waves The Mathematical Theory With Applications (1957) //Institute of Mathematical Sciences New York University, New York .- 609 p.
5. Ватсон Г.Н. Беселевые функции (1949) //М: ИЛ 1949. 276 с.
6. Янке Е., Емде Ф., Леш Ф. Специальные функции, формулы, графики, таблицы (1968) //Перевод с немецкого. М: Наука.- 344 с.
7. Никифоров А.Ф., Уваров В.В. Специальные функции математической физики (1984) //М: Наука.- 344 с.
8. Никифоров А.Ф., Сидорчук В.М., Уваров В.В. Основы теории специальных функции (1974) //М: Наука.- 304 с.
9. Mathews, J. and Walker, R.W.: Mathematical Methods of Physics, Addison-Wesley, Marlo Park, second edition, 1970.
10. Чанишвили А.Г. Безнапорное равномерное движение жидостей в трубопроводах (1947) //Изв. ТНИГЭЙ, №1, Тбилиси.- с. 69-85.
11. Bülow FR. V. Die Leistungfahrigkeit von Flug BachWerkgraben, Kanal und Rohrquerschnitten. Gesundsheits Ingenier, 1927. v. 50. h.262.
12. Еврейнов В.Н. Гидравлика (1939) //Изд. Водный транспорт, М: Л.- 632 с.
13. Шевелёв М.М., Буринская Т.М. Нелинейная динамика неустойчивости Кельвина-Гельмгольца в потоке плазмы конечной ширины (2013) // Физика плазмы. Т. 39.- с. 546.

ԿԼՈՐ ԿՏՐՎԱԾՔՈՎ ՈՉ ՃՆՇՈՒՄԱՅԻՆ ՋՐԱՏԱՐՆԵՐՈՒՄ ՋՐԻ ՇԱՐԺՄԱՆ ԿԱՅՈՒՆՈՒԹՅԱՆ ՄԱՍԻՆ

Գագոշիձե Շ.Ն., Կոդուա Մ.Ա.
Վրաստանի տեխնիկական համալսարան

Դիտարկվում է կլոր կտրվածքով ջրատարում ոչ ճնշումային շարժման կայունությունը: Ալիքային գրգռումների մեթոդի կիրառմամբ, առաջին անգամ, մաթեմատիկորեն հիմնավորվում են գրեթե ամբողջությամբ ջրով լցված կլոր կտրվածքով թունելում կամ ջրատարում շարժման խզվածության պատճառները:

Ստացված տեսական արդյունքները համադրելի են գոյություն ունեցող բնօրինակ և լաբորատոր փորձարարական տվյալների հետ: 92-93% ջրով լցված կլոր կտրվածքով ջրատարում շարժոմը անկայուն է, դիտվում է առաստաղին ջրի ցայտումներ:

Արտածվել է նաև կիսով և փոքր չափով ջրով լցված կլոր կտրվածքով ջրատարում շարժման ասիմտոտիկ հավասարումներ, որոնց որոկական վերլուծությունը ցույց է տալիս, որ այդ պարագայում առաջացող մակերևութային ալիքները կայուն են:

Բանալի բառեր. ոչ ճնշումային շարժում, կլոր կտրվածք, ալիքային գրգռումներ, կայունություն:

ОБ УСТОЙЧИВОСТИ ДВИЖЕНИЯ ВОДЫ В БЕЗНАПОРНЫХ ВОДОВОДАХ КРУГЛОГО ПОПЕРЕЧНОГО СЕЧЕНИЯ

Гагошидзе Ш.Н., Кодуа М.А.
Грузинский технический университет

Рассматривается устойчивость движения жидкости в безнапорных водоводах круглого поперечного сечения. С помощью метода волновых возмущений впервые математически обосновано, почему течения в туннелях или трубопроводах круглого сечения происходит прерывисто, когда они заполнены почти полностью. Полученные теоретические результаты согласуются с существующими экспериментальными и натурными наблюдениями, согласно которым в безнапорных водоводах круглого сечения при их заполнении более чем на 92-93% вода всегда движется прерывисто, со всплесками на потолок, т. е. неустойчиво.

Выводятся также асимптотические уравнения для описания волновых движений в наполовину или в малой степени наполненных каналах круглого поперечного сечения, качественный анализ которых указывает на устойчивость возникших в них поверхностных волн.

Ключевые слова: безнапорное движение, круглое сечение, волновые возмущения, устойчивость.

Submitted on 05.09.2022.
Sent for review on 15.09.2022.
Guaranteed for printing on 14.10.2022.