UDC - 530.145.1

SPHEROIDAL BASIS OF THE GENERALIZED MIK-KEPLER PROBLEM

Mara G. Petrosyan
Shushi University of Technology
7 V.Vagharshyan, Stepanakert, RA
mpetrosyan130@gmail.com
ORCID iD: 0000-0001-5397-4907
Republic of Artsakh

Eleonora V. Avanesyan
Shushi University of Technology
7 V.Vagharshyan, Stepanakert, RA
avanesyane@rambler.ru
ORCID iD: 0000-0002-4443-2353
Republic of Artsakh

https://doi.org/ 10.56243/18294898-2022.3-24

Abstract

Super integrated systems have an extremely important property: they allow the separation of variables in the Hamilton-Jacobi and Schrödinger equations in several orthogonal coordinate systems. The choice of a specific coordinate system is dictated by considerations of convenience, for example, the spectroscopic problem of hydrogen-like systems uses a spherical coordinate system, when considering the Stark effect - a parabolic coordinate system, and in the two-center problem - elongated spheroid coordinates. This abundance of separation of variables in the Schrödinger equation for super integrated systems leads to the problem of interphasic decompositions, i.e. there is a need to move from one wave function to another. The generalized MIC-Kepler problem in spherical coordinates is considered as an explicit form of the additional motion integral and the generalized MIC-Kepler problem in spheroid coordinates is given

$$\hat{\Lambda} = \hat{M} + \frac{R\sqrt{\mu_0}}{\hbar}\hat{\Omega}^{(s)}$$

main function of which is the spheroid basis and three-membered recurrent relations are derived to which the decomposition coefficients of the spheroid basis according to spherical and parabolic bases as well.

Key words: Four-dimensional Schrödinger equation, generalized MIK-Kepler problem, spherical coordinates, spheroid coordinates.

Introduction

A model of a minimally super integral system, a generalized MIC-Kepler problem, was proposed in work and it is described by the Hamiltonian [1].

$$\hat{H} = \frac{1}{2\mu_0}\left(-i\hbar\vec{\nabla} + \frac{e}{c}\vec{A}\right)^2 + \frac{\hbar^2 s^2}{2\mu_0 r^2} - \frac{e^2}{r} + \frac{\lambda_1}{r(r+z)} + \frac{\lambda_2}{r(r-z)}, \qquad (1.1)$$

where λ_1 and λ_2 non-negative constants, and the vector potential $\vec{A} \equiv \vec{A}^{(+)}$ is defined by the expression where

$$\vec{A}^{(\pm)} = \frac{g}{r(r \mp z)}(\pm y, \mp x, 0).$$

Here the vector potentials $\vec{A}^{(\pm)}$ correspond to the Dirac monopole with a magnetic charge [2].

$$g = \frac{\hbar c s}{e}, \quad \text{where} \quad s = 0, \pm\frac{1}{2}, \pm 1, \ldots,$$

and with the singularity axis at $z > 0$ and $z < 0$ respectively.

It is easy to notice that the vector potentials $\vec{A}^{(+)}$ and $\vec{A}^{(-)}$ are interconnected by a calibration transformation

$$\vec{A}^{(-)} = \vec{A}^{(+)} + grad\, f, \quad \text{where} \quad f = 2g \arctan\frac{y}{x},$$

and the intensity of the magnetic field created by dion has the form

$$\vec{B} = rot\, \vec{A}^{(\pm)} = g\frac{\vec{r}}{r^3}.$$

Further, we will carry out all calculations only for the vector potential $\vec{A}^{(+)}$ and for brevity in the future we will omit the sign $(+)$.

The variables in the Shrödinger equation for the generalized MIC-Kepler problem are separated in spherical, parabolic and elongated spheroid coordinates [3].

Conflict setting

The spherical basis of the generalized task of MIK-Kelper [4] has the following expression form

$$\psi_{njm}^{(s)}\left(r,\theta,\varphi;\delta_1^{(s)};\delta_2^{(s)}\right) = R_{nj}^{(s)}\left(r;\delta_1^{(s)};\delta_2^{(s)}\right) Z_{jm}^{(s)}\left(\theta,\varphi;\delta_1^{(s)};\delta_2^{(s)}\right), \qquad (2.1)$$

where normalized by the conditions

$$\int_0^{\pi}\int_0^{2\pi} \sin\theta\, Z_{j'm'}^{(s)*}\left(\theta,\varphi;\delta_1^{(s)},\delta_2^{(s)}\right) Z_{jm}^{(s)}\left(\theta,\varphi;\delta_1^{(s)},\delta_2^{(s)}\right) d\theta\, d\varphi = \delta_{jj'}\,\delta_{mm'}, \qquad (2.2)$$

$$\int_0^{\infty} r^2 \left[R_{nj}^{(s)}\left(r;\delta_1^{(s)},\delta_2^{(s)}\right)\right]^2 dr = 1 \qquad (2.3)$$

the angular and radial wave functions have the following form

$$Z_{jm}^{(s)}\left(\theta,\varphi;\delta_1^{(s)};\delta_2^{(s)}\right)=N_{jm}\left(\delta_1^{(s)};\delta_2^{(s)}\right)\left(\sin\frac{\theta}{2}\right)^{m_1}\left(\cos\frac{\theta}{2}\right)^{m_2}P_{j-m_+}^{(m_2,m_1)}(\cos\theta)e^{i(m+s)\varphi}, \qquad (2.4)$$

$$R_{nj}^{(s)}\left(r;\delta_1^{(s)};\delta_2^{(s)}\right)=C_{nj}\left(\delta_1^{(s)};\delta_2^{(s)}\right)(2\varepsilon r)^{j+\frac{\delta_1^{(s)}+\delta_2^{(s)}}{2}}e^{-\varepsilon r}F\left(-n+j+1;2j+\delta_1^{(s)}+\delta_2^{(s)};2\varepsilon r\right), \quad (2.5)$$

here

$$m_1=|m+s|+\delta_1^{(s)}=\sqrt{(m+s)^2+\frac{4\mu_0\lambda_1}{\hbar^2}}, \qquad (2.6)$$

$$m_2=|m-s|+\delta_2^{(s)}=\sqrt{(m-s)^2+\frac{4\mu_0\lambda_2}{\hbar^2}}, \qquad (2.7)$$

$$N_{jm}\left(\delta_1^{(s)},\delta_2^{(s)}\right)=(-1)^{\frac{m-s+|m-s|}{2}}\sqrt{\frac{\left(2j+\delta_1^{(s)}+\delta_2^{(s)}+1\right)(j-m_+)!\,\Gamma\left(j+m_++\delta_1^{(s)}+\delta_2^{(s)}+1\right)}{4\pi\,\Gamma\left(j+m_-+\delta_1^{(s)}+1\right)\Gamma\left(j-m_-+\delta_2^{(s)}+1\right)}} \qquad (2.8)$$

$$C_{nj}\left(\delta_1^{(s)},\delta_2^{(s)}\right)=\frac{2\varepsilon^2\sqrt{r_0}}{\Gamma\left(2j+\delta_1^{(s)}+\delta_2^{(s)}+2\right)}\sqrt{\frac{\Gamma\left(n+j+\delta_1^{(s)}+\delta_2^{(s)}+1\right)}{(n-j-1)!}}, \qquad (2.9)$$

$$\varepsilon=\sqrt{-\frac{2\mu_0 E}{\hbar^2}}=\frac{1}{r_0\left(n+\frac{\delta_1^{(s)}+\delta_2^{(s)}}{2}\right)}, \qquad (2.10)$$

the $m_\pm$ is defined by ratio (2.6).

The energy (E) Eigen values of the generalized MIC-Kepler task are determined by the following formula

$$E\equiv E_n^{(s)}=-\frac{\mu_0 e^4}{2\hbar^2\left(n+\frac{\delta_1^{(s)}+\delta_2^{(s)}}{2}\right)^2}. \qquad (2.11)$$

The quantum numbers n, j and m express the following formulas: $n=|s|+1,\ |s|+2,\ldots,$ $j=m_+,\ m_++1,\ldots,\ n-1$, a $m=-j,\ -j+1,\ldots,\ j-1,\ j$.

Quantum numbers j and m characterize the total moment of the system and its projection on axis. For (semi) integers s, j and m are also (semi) integers.

It is important to note that the spherical wave function of the generalized MIK-Kepler task is an Eigen function of the system of commuting operators $\{\hat{H},\hat{M},\hat{J}_z\}$ and carries the following spectral tasks:

$$\hat{H}\,\psi_{njm}^{(s)}\left(r,\theta,\varphi;\delta_1^{(s)};\delta_2^{(s)}\right)=-\frac{\mu_0 e^4}{2\hbar^2\left(n+\frac{\delta_1^{(s)}+\delta_2^{(s)}}{2}\right)^2}\psi_{njm}^{(s)}\left(r,\theta,\varphi;\delta_1^{(s)};\delta_2^{(s)}\right), \qquad (2.12)$$

$$\hat{M}\,\psi^{(s)}_{njm}\left(r,\theta,\varphi;\delta_1^{(s)};\delta_2^{(s)}\right)=\left(j+\frac{\delta_1^{(s)}+\delta_2^{(s)}}{2}\right)\left(j+\frac{\delta_1^{(s)}+\delta_2^{(s)}}{2}+1\right)\psi^{(s)}_{njm}\left(r,\theta,\varphi;\delta_1^{(s)};\delta_2^{(s)}\right) \qquad (2.13)$$

$$\hat{J}_z\,\psi^{(s)}_{njm}\left(r,\theta,\varphi;\delta_1^{(s)};\delta_2^{(s)}\right)=-\left(s+i\frac{\partial}{\partial\varphi}\right)\psi^{(s)}_{njm}\left(r,\theta,\varphi;\delta_1^{(s)};\delta_2^{(s)}\right)=m\,\psi^{(s)}_{njm}\left(r,\theta,\varphi;\delta_1^{(s)};\delta_2^{(s)}\right) \qquad (2.14)$$

$$\hat{M}=\hat{J}^2+\frac{2\mu_0}{\hbar^2}\left(\frac{\lambda_1}{1+\cos\theta}+\frac{\lambda_2}{1-\cos\theta}\right). \qquad (2.15)$$

In the following case $\hat{J}^2$ is the square of the total momentum operator

$$\hat{\vec{J}}=\vec{r}\times\left(-i\vec{\nabla}+\frac{e}{\hbar c}\vec{A}\right)+s\frac{\vec{r}}{r}. \qquad (2.16)$$

The total wave function of the generalized MIC-Kepler system, considering normalization to unity in parabolic coordinates, can be represented as [5]

$$\psi^{(s)}_{n_1 n_2 m}\left(\mu,\nu,\varphi;\delta_1^{(s)},\delta_2^{(s)}\right)=\varepsilon^2\sqrt{\frac{r_0}{\pi}}\;\Phi^{(s)}_{n_1 m_1}\left(\mu;\delta_1^{(s)}\right)\Phi^{(s)}_{n_2 m_2}\left(\nu;\delta_2^{(s)}\right)e^{i(m+s)\varphi}, \qquad (2.17)$$

where

$$\Phi^{(s)}_{n_i m_i}\left(t_i,\delta_i^{(s)}\right)=\sqrt{\frac{\Gamma(n_i+m_i+1)}{(n_i)!}}\,\frac{e^{-\frac{\varepsilon t_i}{2}}\left(\varepsilon t_i\right)^{\frac{m_i}{2}}}{\Gamma(m_i+1)}F\left(-n_i;\,m_i+1;\,\varepsilon t_i\right), \qquad (2.18)$$

What is more is that $i=1,2$ and $t_1=\mu$ a $t_2=\nu$. Here n_1 and n_2 are equal and non-negative integers.

$$n_1=-\frac{|m+s|+\delta_1^{(s)}+1}{2}+\frac{\sqrt{\mu_0}}{2\varepsilon\hbar}\Omega^{(s)}+\frac{1}{2r_0\,\varepsilon}, \qquad (2.19)$$

$$n_2=-\frac{|m-s|+\delta_2^{(s)}+1}{2}-\frac{\sqrt{\mu_0}}{2\varepsilon\hbar}\Omega^{(s)}+\frac{1}{2r_0\,\varepsilon}. \qquad (2.20)$$

Here $\Omega^{(s)}$ is the separation constant in parabolic coordinates. Now, taking into account the last examples, we understand that the parabolic quantum numbers are related to the principal quantum number n, in case of the MIK-Kepler task it goes the following way

$$n=n_1+n_2+m_++1. \qquad (2.21)$$

In the end, we note that the (2.17)parabolic basis is an Eigen function of both the Hamilton (1.1) operator and the -component of the total moment (1.16), and the operator

$$\hat{\Omega}^{(s)} = \hat{I}_z + \frac{\mu_0}{\hbar^2}\left[\lambda_1 \frac{r-z}{r(r+z)} - \lambda_2 \frac{r+z}{r(r-z)}\right], \qquad (2.22)$$

where $\hat{I}_z$ z- components are analogous to the Laplace-Runge-Lenz-Pauli vector [6]. The Eeigen value of the operator is the parabolic separation constant, whose explicit form is

$$\Omega^{(s)} = \frac{\hbar\varepsilon}{\sqrt{\mu_0}}\left(n_1 - n_2 + m_- + \frac{\delta_1^{(s)} - \delta_2^{(s)}}{2}\right). \qquad (2.23)$$

Taking into account the ratios (2.10), the expression (2.23) is obtained from ratios (2.19) and (2.20).

In the end, it is important to note that all the formulas expressed above indicate that they can be obtained from the corresponding formulas for the MIC-Kepler task using the following ansatzs:

$$n \to n + \frac{\delta_1^{(s)} + \delta_2^{(s)}}{2}, \qquad j \to j + \frac{\delta_1^{(s)} + \delta_2^{(s)}}{2},$$
$$|m+s| \to |m+s| + \delta_1^{(s)} \equiv m_1, \qquad |m-s| \to |m-s| + \delta_2^{(s)} \equiv m_2 \quad . \qquad (2.24)$$

Then the mutual expansions of the parabolic and spherical basis of the generalized MIK-Kepler problem [7] will have the following form

$$\psi^{(s)}_{n_1 n_2 m}\left(\mu,\nu,\varphi;\, \delta_1^{(s)},\delta_2^{(s)}\right) = \sum_{j=m_+}^{n-1} W^{j}_{n n_1 m s}\left(\delta_1^{(s)},\delta_2^{(s)}\right)\psi^{(s)}_{njm}\left(r,\theta,\varphi;\, \delta_1^{(s)},\delta_2^{(s)}\right),$$

$$\psi^{(s)}_{njm}\left(r,\theta,\varphi;\, \delta_1^{(s)},\delta_2^{(s)}\right) = \sum_{n_1=0}^{n-m_+-1} \widetilde{W}^{j}_{n n_1 m s}\left(\delta_1^{(s)},\delta_2^{(s)}\right)\psi^{(s)}_{n_1 n_2 m}\left(\mu,\nu,\varphi;\, \delta_1^{(s)},\delta_2^{(s)}\right)$$

where, according to formulas

$$n \to n + \frac{\delta_1^{(s)} + \delta_2^{(s)}}{2}, \qquad j \to j + \frac{\delta_1^{(s)} + \delta_2^{(s)}}{2},$$

$$|m+s| \to |m+s| + \delta_1^{(s)} \equiv m_1, \qquad |m-s| \to |m-s| + \delta_2^{(s)} \equiv m,$$

$$W^{j}_{n n_1 m s} = (-1)^{n_1 + \frac{m-s+|m-s|}{2}} C^{j,m_+}_{\frac{n_1+n_2+|m-s|}{2},\frac{n_2-n_1+|m-s|}{2};\frac{n_1+n_2+|m+s|}{2},\frac{n_1-n_2+|m+s|}{2}}$$

and

$$\widetilde{W}^{n_1}_{n j m s} = (-1)^{n_1 + \frac{m-s+|m-s|}{2}} C^{j,m_+}_{\frac{n-m_--1}{2},\frac{n-m_--1}{2}-n_1;\frac{n+m_--1}{2},\frac{m_++|m+s|-n+1}{2}+n_1}$$

for coefficients $W^{j}_{n n_1 m s}\left(\delta_1^{(s)},\delta_2^{(s)}\right)$ and $\widetilde{W}^{j}_{n n_1 m s}\left(\delta_1^{(s)},\delta_2^{(s)}\right)$ we will obtain

$$W_{nn_1ms}^{j}\left(\delta_1^{(s)},\delta_2^{(s)}\right)=(-1)^{n_1+\frac{m-s+|m-s|}{2}}\, C^{j+\frac{\delta_1^{(s)}+\delta_2^{(s)}}{2},\frac{m_1+m_2}{2}}_{\frac{n_1+n_2+m_2}{2},\frac{n_2-n_1+m_2}{2};\frac{n_1+n_2+m_1}{2},\frac{n_1-n_2+m_1}{2}},$$

$$\widetilde{W}_{nn_1ms}^{j}\left(\delta_1^{(s)},\delta_2^{(s)}\right)=(-1)^{n_1+\frac{m-s+|m-s|}{2}}\, C^{j,m_+}_{\frac{n-m_-+\delta_2^{(s)}-1}{2},\frac{n-m_-+\delta_2^{(s)}-1}{2}-n_1;\frac{n+m_-+\delta_1^{(s)}-1}{2},\frac{m_++|m+s|+\delta_1^{(s)}-n+1}{2}+n_1},$$

which, precisely till the phase factor $(-1)^{n_1+\frac{m-s+|m-s|}{2}}$, coincides with the Clebsch-Gordan coefficients of the group $SU(2)$, continued by indices into the phase of real numbers [8].

Research results

In elongated spheroid coordinates ξ,η,φ, which are defined as follows

$$x=\frac{R}{2}\sqrt{(\xi^2-1)(1-\eta^2)}\cos\varphi,\qquad y=\frac{R}{2}\sqrt{(\xi^2-1)(1-\eta^2)}\sin\varphi,\qquad z=\frac{R}{2}(\xi\eta+1),\quad(3.1)$$

where $\xi\in[1,\infty)$, $\eta\in[-1,1]$, $\varphi\in[0,2\pi)$, $R\in[0,\infty)$,

Differential elements of length, volume and Laplace operator have the form

$$dl^2=\frac{R^2}{4}\left[(\xi^2-\eta^2)\left(\frac{d\xi^2}{\xi^2-1}+\frac{d\eta^2}{1-\eta^2}\right)+(\xi^2-1)(1-\eta^2)d\varphi^2\right],$$

$$dv=\frac{R^3}{8}(\xi^2-\eta^2)d\xi\,d\eta\,d\varphi,\qquad(3.2)$$

$$\Delta=\frac{4}{R^2(\xi^2-\eta^2)}\left[\frac{\partial}{\partial\xi}(\xi^2-1)\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\eta}(1-\eta^2)\frac{\partial}{\partial\eta}\right]+\frac{4}{R^2(\xi^2-1)(1-\eta^2)}\frac{\partial^2}{\partial\varphi^2}.$$

The parameter is an interfacial distance and when elongated spheroid coordinates pass into spherical and parabolic coordinates, respectively we have the formula [9].

The spheroid basis of the generalized MIC-Kepler problem is the Eigen function of the operator

$$\hat{\Lambda}=\hat{M}+\frac{R\sqrt{\mu_0}}{\hbar}\hat{\Omega}^{(s)},\qquad(3.3)$$

I.e.

$$\hat{\Lambda}\psi_{nqm}^{(s)}\left(\xi,\eta,\varphi;R,\delta_1^{(s)},\delta_2^{(s)}\right)=\Lambda_q\left(R,\delta_1^{(s)},\delta_2^{(s)}\right)\psi_{nqm}^{(s)}\left(\xi,\eta,\varphi;R,\delta_1^{(s)},\delta_2^{(s)}\right),\qquad(3.4)$$

where the operators $\hat{M}$ and $\hat{\Omega}^{(s)}$ are defined by expressions (2.15) and (2.22), the index q numbers the Eigen values of the operator $\hat{\Lambda}$ and changes in the area $0\le q\le n-m_+-1$. The spheroid basis of the generalized MIC-Kepler problem [10], at a fixed value of energy, can be represented as a quantum mixture of spherical and parabolic bases:

$$\psi^{(s)}_{nqm}\left(\xi,\eta,\varphi;R,\delta^{(s)}_1,\delta^{(s)}_2\right)=\sum_{j=m_+}^{n-1}U^{j}_{nqms}\left(R,\delta^{(s)}_1,\delta^{(s)}_2\right)\psi^{(s)}_{njm}\left(r,\theta,\varphi;\ \delta^{(s)}_1,\delta^{(s)}_2\right), \quad (3.5)$$

$$\psi^{(s)}_{nqm}\left(\xi,\eta,\varphi;R,\delta^{(s)}_1,\delta^{(s)}_2\right)=\sum_{n_1=0}^{n-m_+-1}V^{n_1}_{nqms}\left(R,\delta^{(s)}_1,\delta^{(s)}_2\right)\psi^{(s)}_{n_1n_2m}\left(\mu,\nu,\varphi;\ \delta^{(s)}_1,\delta^{(s)}_2\right), \quad (3.6)$$

Decomposition coefficients (3.5), (3.6) U^{j}_{nqms} и $V^{n_1}_{nqms}$ are determined from the following three-member recurrent relations

$$\left[\left(j+\frac{\delta^{(s)}_1+\delta^{(s)}_2}{2}\right)\left(j+\frac{\delta^{(s)}_1+\delta^{(s)}_2}{2}+1\right)+\frac{R\left(m_1+m_2\right)\left(m_1-m_2\right)}{r_0\left(2j+\delta^{(s)}_1+\delta^{(s)}_2\right)\left(2j+\delta^{(s)}_1+\delta^{(s)}_2+2\right)}-\right.$$

$$\left.-\Lambda_q\left(R,\delta^{(s)}_1,\delta^{(s)}_2\right)\right]U^{j}_{nqms}=\frac{2R}{r_0\left(2n+\delta^{(s)}_1+\delta^{(s)}_2\right)}\left[A^{j+1}_{nm}\,U^{j+1}_{nqms}+A^{j}_{nm}\,U^{j-1}_{nqms}\right], \quad (3.7)$$

$$\left[\left(n_1+|m+s|+\delta^{(s)}_1\right)\left(n_2+|m-s|+\delta^{(s)}_2+1\right)+m_-\left(m_-+\delta^{(s)}_1-\delta^{(s)}_2-1\right)+n_2\left(n_1+1\right)+\right.$$

$$\left.+\frac{1}{4}\left(\delta^{(s)}_1-\delta^{(s)}_2\right)\left(\delta^{(s)}_1-\delta^{(s)}_2-2\right)+\frac{R\left(2n_1-2n_2+m_1-m_2\right)}{r_0\left(2n+\delta^{(s)}_1+\delta^{(s)}_2\right)}-\Lambda_q\left(R,\delta^{(s)}_1,\delta^{(s)}_2\right)\right]V^{n_1}_{nqms}=$$

$$=\sqrt{n_2\left(n_1+1\right)\left(n_1+|m+s|+\delta^{(s)}_1+1\right)\left(n_2+|m-s|+\delta^{(s)}_2\right)}\,V^{n_1+1}_{nqms}+$$

$$+\sqrt{n_1\left(n_2+1\right)\left(n_1+|m+s|+\delta^{(s)}_1\right)\left(n_2+|m-s|+\delta^{(s)}_2+1\right)}\,V^{n_1-1}_{nqms}, \quad (3.8)$$

where

$$A^{j}_{nm}=\frac{2\sqrt{\left(j-m_+\right)\left(n-j\right)\left(j+m_++\delta^{(s)}_1+\delta^{(s)}_2\right)}}{2j+\delta^{(s)}_1+\delta^{(s)}_2}\times$$

$$\times\left[\frac{\left(j+m_-+\delta^{(s)}_1\right)\left(j-m_-+\delta^{(s)}_2\right)\left(n+j+\delta^{(s)}_1+\delta^{(s)}_2\right)}{\left(2j+\delta^{(s)}_1+\delta^{(s)}_2-1\right)\left(2j+\delta^{(s)}_1+\delta^{(s)}_2+1\right)}\right]^{\frac{1}{2}} \quad (3.9)$$

recurrent ratios (3.8) and (3.9) need to be solved together with the conditions

$$\sum_{j=m_+}^{n-1}\left|U^{j}_{nqms}\left(R,\delta^{(s)}_1,\delta^{(s)}_2\right)\right|^2=1, \qquad \sum_{n_1=0}^{n-m_+-1}\left|V^{n_1}_{nqms}\left(R,\delta^{(s)}_1,\delta^{(s)}_2\right)\right|^2=1. \quad (3.10)$$

Conclusion

Spheroid coordinates are a natural means of studying many problems of mathematical physics. In quantum mechanics, these coordinates are used to describe the behavior of a charged particle in the field of two Coulomb centers. The distance between the centers characterizes spheroid coordinates and is included in the expression for the energy spectrum, i.e. it has a dynamic meaning. If the charge of one of the centers is put equal to zero, the problem goes to a single-center one and the parameter becomes purely kinematic. However, the mathematical structure of spheroid equations remains largely the same, since energy is included in both the radial and angular equations. In this regard, the spheroid analysis of the generalized MIC-Kepler problem acquires the meaning of the first step towards the study of the two-centers problem of the generalized MIC-Kepler problem. The integral of motion responsible for the separation of variables in a spheroid coordinate system is obtained, and three-term recurrence relations are derived being complied with by the coefficients of the expansion of the spheroid basis.

References

1. Mardoyan L.G. (2003) The generalized MIC-Kepler system //J. Math. Phys., 44.- p.4981-4987.
2. Dirac P.A.M. (1931) Quantized singularities in the Electromagnetic Field //Proc. Roy. Soc., A133.- p. 60-72.
3. Mardoyan L.G. (2005) Spheroid analysis of the generalized MIC-Kepler system //«Nuclear physics», 68.- p. 4981-1808-1816.
4. Nersessian, A. Generalizations of MICZ-Kepler system. Phys. Atom. Nuclei 73, 489–493 (2010). https://doi.org/10.1134/S1063778810030117
5. Meng G., Generalized MICZ-Kepler problems and unitary highest weight modules, II, (2010)// J. Lond. Math. Soc. 81, 663–678. https://doi.org/10.1112/jlms/jdq019
6. Galikova V., Presnajder P. et al, Laplace-Runge-Lenz vector in quantum mechanics in non-commuting space , (2013) //JMP,54,122106. https://doi.org/10.1063/1.4835615
7. Kibler M., Mardoyan L.G., Pogosyan G.S. (1994) On a Generalized Kepler-Coulomb System: Inter based Expansions. Int. J. Quant. Chem., 52.- p. 1301-1316. file:///C:/Users/HP/Downloads/M.%20Kilber,%20L.%20G.%20Mardoyan,%20G.%20S.%20Pogosyan.pdf
8. Chen G. Clebsch-Gordan coefficients of discrete groups in subgroup bases, (2018) //Int. J. Mod. Phys. A, 33, 1850055. https://doi.org/10.1142/S0217751X18500550
9. Sege G. Orthogonal bineral numerals (1962) //M: «FizMatGiz», 500p.
10. Meng G. Generalized Kepler problems. I. Without magnetic charges, (2013). // J. Math. Phys. 54, 012109. https://doi.org/10.1063/1.4775343

References

1. Mardoyan L.G. (2003) The generalized MIC-Kepler system //J. Math. Phys., 44.- p.4981-4987.
2. Dirac P.A.M. (1931) Quantized singularities in the Electromagnetic Field //Proc. Roy. Soc., A133.- p. 60-72.
3. ardoyan L.G. (2005) Spheroid analysis of the generalized MIC-Kepler system //«Nuclear physics», 68.- p. 4981-1808-1816.
4. Nersessian, A. Generalizations of MICZ-Kepler system. Phys. Atom. Nuclei 73, 489–493 (2010). https://doi.org/10.1134/S1063778810030117
5. Meng G., Generalized MICZ-Kepler problems and unitary highest weight modules, II, (2010)// J. Lond. Math. Soc. 81, 663–678. https://doi.org/10.1112/jlms/jdq019
6. Galikova V., Presnajder P. et al, Laplace-Runge-Lenz vector in quantum mechanics in non-commuting space, (2013) //JMP,54,122106. https://doi.org/10.1063/1.4835615

7. Kibler M., Mardoyan L.G., Pogosyan G.S. (1994) On a Generalized Kepler-Coulomb System: Inter based Expansions. Int. J. Quant. Chem., , 52.- p. 1301-1316. file:///C:/Users/HP/Downloads/M.%20Kilber,%20L.%20G.%20Mardoyan,%20G.%20S.%20Pogosyan.pdf
8. Chen G. Clebsch-Gordan coefficients of discrete groups in subgroup bases, (2018) //Int. J. Mod. Phys. A, 33, 1850055. https://doi.org/10.1142/S0217751X18500550
9. Сеге Г. (1962) Ортогональные многочлены //М: «ФизМатГиз», 500с.
10. Meng G. Generalized Kepler problems. I. Without magnetic charges, (2013). // J. Math. Phys. 54, 012109. https://doi.org/10.1063/1.4775343

ԸՆԴՀԱՆՐԱՑՎԱԾ ՄԻԿ-ԿԵՊԼԵՐԻ ԽՆԴՐԻ ՍՖԵՐՈԻԴԱԼ ԲԱԶԻՍԸ

Պետրոսյան Մ.Գ., Ավանեսյան Է.Վ.
Շուշիի տեխնոլոգիական համալսարան

Սուպերինտեգրվող համակարգերը ունեն շատ կարևոր մի հատկություն՝ Համիլտոն Յակոբիի և Շրեդինգերի հավասարումները փոփոխականների բաժանման մեթոդով ճշգրիտ լուծվում են մի քանի օրթոգոնալ կոորդինատային համակարգերում: Կոնկրետ կոորդինատային համակարգի ընտրությունը թելադրված է հարմարության տեսանկյունից, օրինակ՝ ջրածնանման համակարգերի սպեկտրոսկոպիկ խնդրի դեպքում օգտագործվում է սֆերիկ կոորդինատային համակարգը, Շտարկի էֆեկտը դիտարկելիս՝ պարաբոլիկ կոորդինատային համակարգը և երկկենտրոն խնդիրը դիտարկելիս՝ սֆերոիդային կոորդինատները: Շրեդինգերի հավասարման մեջ փոփոխականների բաժանման նման առատությունը սուպերինտեգրելի համակարգերի համար հանգեցնում է միջբազիսային վերլուծության խնդրին, այսինքն՝անհրաժեշտություն է առաջանում մի ալիքային ֆունկցիայից անցնել մեկ այլ ալիքային ֆունկցիայի:

Դիտարկված է ընդհանրացված ՄԻԿ-Կեպլերի խնդիրը սֆերիկ կոորդինատներում: Ստացվել է ընդհանրացված ՄԻԿ-Կեպլերի խնդրի լրացուցիչ շարժման ինտեգրալի բացահայտ տեսքը սֆերոիդալ կոորդինատներում՝

$$\hat{\Lambda} = \hat{M} + \frac{R\sqrt{\mu_0}}{\hbar}\hat{\Omega}^{(s)},$$

որի սեփական ֆունկցիան սֆերոիդալ բազիսն է, և դուրս են բերվել եռանդամ ռեկուրենտ առընչություններ, որոնց բավարարում են սֆերոիդալ բազիսը ըստ սֆերիկ և պարաբոլիկ բազիսների վերլուծության գործակիցները:

Բանալի բառեր. Շրեդինգերի քառաչափ հավասարում, ընդհանրացված ՄԻԿ-Կեպլերի խնդիր, սֆերիկ կոորդինատներ, սֆերոիդալ կոորդինատներ:

СФЕРОИДАЛЬНЫЙ БАЗИС ОБОБЩЕННОЙ ЗАДАЧИ МИК-КЕПЛЕРА

Петросян М.Г., Аванесян Э.В.
Шушинский технологический университет

Суперинтегрируемые системы обладают исключительно важным свойством: они допускает разделение переменных в уравнениях Гамильтона-Якоби и Шредингера в нескольких ортогональных системах координат. Выбор конкретной системы координат диктуется соображениями удобства, например, в спектроскопической задаче водородоподобных систем используется сферическая система координат, при рассмотрении эффекта Штарка – параболическая система координат, а в двухцентровой задаче – вытянутые сфероидальные координатаы. Такое обилие разделения переменных в уравнении Шредингера для суперинтегрируемых систем приводит к задаче о межбазисных разложений, т.е. возникает необходимость перехода от одной волновой функции к другой.

Рассмотрена обобщенная задача МИК-Кеплера в сферических координатах. Приведен явный вид добавочного интеграла движения обобщенной задачи МИК-Кеплера в сфероидальных координатах

$$\widehat{\Lambda} = \widehat{M} + \frac{R\sqrt{\mu_0}}{\hbar}\widehat{\Omega}^{(s)},$$

собственней функцией которого является сфероидальный базис, и выведены трехчленные рекуррентные соотношения, которым подчиняются коэффициенты разложений сфероидального базиса по сферическому и параболическому базисам.

Ключевые слова: четырехмерное уравнение Шредингера, обобщенная задача МИК-Кеплера, сферические координаты, сфероидальные координаты.

Submitted on 18.05.2022.
Sent for review on 27.05.2022.
Guaranteed for printing on 14.10.2022.