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APPLICATION OF ITERATIVE INCREMENTAL REDUCTION METHOD TO SOLVE THE PROBLEM OF OPTIMAL DESIGN OF A CYLINDRICAL PLATFORM DURING ITS BENDING

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Abstract

In this paper we suggest a model for solving the optimization problem which allows the use of a method of unconstrained minimization Nelder-Mead in complying with the restrictions in the form of equations and inequalities.

The application of this model is illustrated by the optimal design of a cylindrical panels made of isotropic material.

The optimal values of the geometrical parameters of the plate to ensure minimum deflection at constant weight equal to the weight of the plate thickness is constant. The results are also compared in order to additionally check their accuracy. The problem is also solved by the direct search method for comparing the latter with the previous.

Key words: optimization, maximization, isotropic material, load, bending, panel, bending, iterative.

Statement of problem optimization

Many problems of optimal design of ribbed plates made of an isotropic material are reduced to a nonlinear programming problem under constraints in the form of equalities and inequalities. In general, this problem is formulated as follows:

To find

$$\min_{\bar{u}} \max_{\bar{v}} f(\bar{u},\bar{v}), \quad \bar{u} = \{u_1, u_2, ..., u_n\},\ \bar{v} = \{v_1, v_2, ..., v_n\}$$

with restrictions

$$H_i(\bar{u},\bar{v}) = 0,\ i = 1,2,...,m,$$

$$g_j(\bar{u},\bar{v}) \geq 0,\quad j = m+1, m+2,...,r,$$

where $f(\bar{u},\bar{v})$ - is the objective function; $\bar{u},\bar{v}$ - are the control vectors.

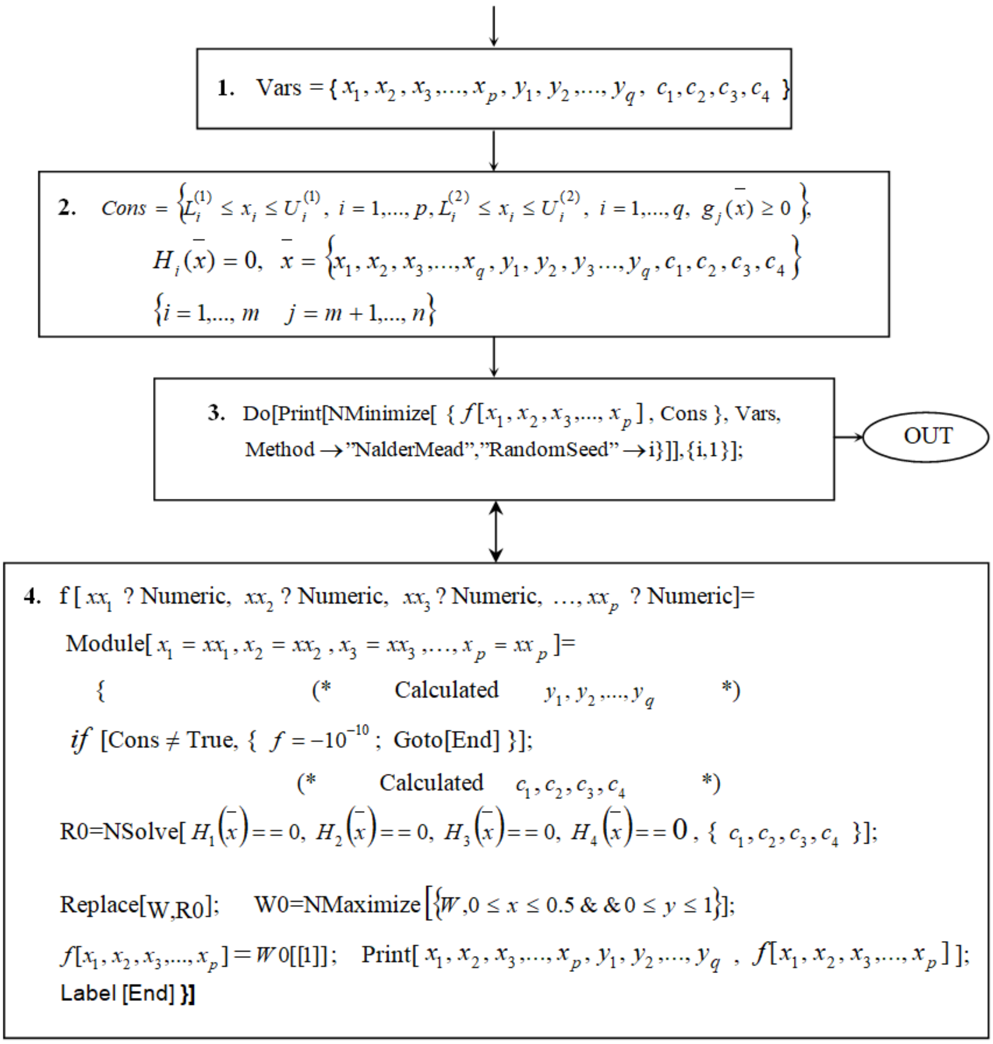
The complexity of solving a problem increases geometrically as the number of variables or constraints increases. In the problem preparation phase, it is advisable to modify the model to reduce the number of constraints, especially nonlinear and variable ones. The size and number of constraints in the form of equations can significantly be reduced by explicitly or indirectly solving some of them and eliminating the variables with the resulting solutions. This leaves a number of constraints that cannot be directly addressed for one or more variables. In this case, two approaches are possible. These constraints are solved iteratively and incrementally with respect to the variables, or the remaining equations are clearly considered constraints in the form of equations.

The solution to the proposed minimum problem is achieved by solving the problems of successive reduction and increase of constraints.

The basic idea of the proposed models consists in the volume, which is the solution to the problem of common tasks by dividing the intermediate into two groups. The variables $x_1, x_2, ..., x_p$ join in one group, the values of which is impossible to exclude from the corresponding constraints, and in the other – the variables $y_1, y_2, ..., y_q, c_1, c_2, c_3, c_4$, the values of which are relatively easy to calculate. Both problems are solved separately, ensuring their connection by carrying out the corresponding coordinating calculations.

The minimization problem is solved using the Nelder-Mead method [4]. Moreover, in the proposed method, when constructing a simplex, at each stage of minimization the vertex of the polyhedron is excluded where the constraints are violated.

The block diagram of the program is given with the help of which numerical calculations are performed.



In the block diagram:
1 - excretion of variable parameters,
2 - formulation of restrictions,
3 - selection of the Nelder-Mead method, where the objective function has a parametric form. According to the Nelder-Mead method, when searching for the minimum of the objective function $f[x_1, x_2, ..., x_p]$, where $x \in E^p$] a polygon is constructed with p+1 vertex. A new vertex is constructed using the stretching, compression and reduction procedures, and the value of the objective function is calculated at each vertex using the module (point 4):

$$\max_{\bar{z}} f[x_1, x_2, ..., x_p], \text{ где } \bar{z} = \{y_1, y_2, ..., y_q, c_1, c_2, c_3, c_4\}.$$

The iterative incremental process ends when the vertices of the simplex and the values of the function calculated in them satisfy some convergence conditions when compared with the previous iteration,
4 - objective function formulation in parametric form (Module $f[x_1, x_2, ..., x_p]$), where the maximization problem is solved. At each vertex of the simplex, two operations are carried out sequentially: the inclusion of vertices in the admissible region and the exclusion of vertices if any additional condition is violated where $f[x_1, x_2, ..., x_p] = -10^{-10}$.

As an application of the proposed model, the problem of optimal design of a shallow cylindrical shell of an open profile of piecewise constant thickness is solved under the action of internal pressure during bending. The mathematical model of the problem of determining the stress-strain state of the shell is described by differential equations with respect to potential functions for each of the constituent parts of the shell, conjugation conditions on the lines of their separation and boundary conditions on its contour. The determination of the optimal parameters of the panel is reduced to a nonlinear programming problem, which is solved by the deformable polyhedron method [4] in combination with the direct search method [4] and using the parallel computing package in the WolframMatematika package environment [4].

The issues of optimal design of plates and shells of piecewise constant thickness were studied in [1-3].

Formulation of the problem

A shallow cylindrical shell with dimensions is considered in plan $2L \times b$, hanging support on the sides $y = 0$ and $y = b$ and rigidly fixed at the edges $x = \pm L$, under the influence of the internal pressure $q(y)$. It is assumed that on the site $-a \le x \le a$ the staired shell has thickness of h_2, and on the sites $-L \le x \le -a$ and $a \le x \le L$ - thickness h_1 (fig. 1).

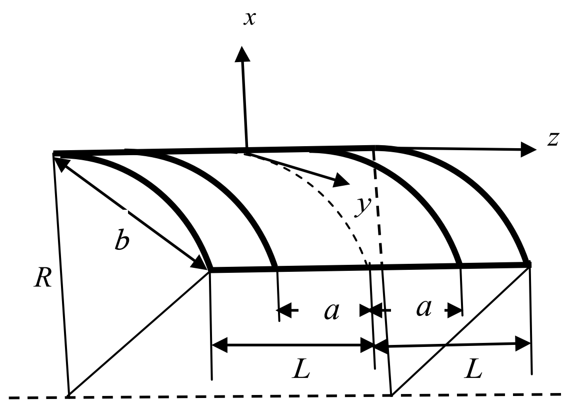


Fig. 1 Design scheme of the shell

The task is to determine the optimal values of the parameters of a, h_1, h_2, providing the minimum value of the greatest deflection of the shell at constant weight, equal to the weight of the shell of constant thickness h_0, and given overall dimensions $\xi = 2L/b$.

Determination of the stress-strain state of the shell

The problem of determining the stress-strain state of a shell of piecewise constant thickness is solved for each of its regions $(p=1,\ 2)$, corresponding to the thicknesses and, with the satisfaction of the conjugation conditions on the line of their division. In this case, in the form of symmetry, the right half of the shell is considered ($x \geq 0$) . According to the theory of very shallow shells, the problem [1] is reduced to determining for each of the sections of the shell $(p=1,\ 2)$ potential functions $\Phi_p(x, y)$, satisfying the equations

$$\frac{D^{(p)}}{E_p h_p}\nabla^4\nabla^4\Phi_p + \frac{1}{R^2}\frac{\partial^4\Phi_p}{\partial x^4} = q,\ (p=1,\ 2) \tag{1}$$

where: $D^{(p)} = \dfrac{E_p h_p^3}{12(1-\nu_p^2)}$ - stiffness of the constituent sections of the shell, E_p and ν_p - elastic constants of materials,

$$\nabla^8 = \frac{\partial^8}{\partial x^8} + 4\frac{\partial^8}{\partial x^6\partial y^2} + 6\frac{\partial^8}{\partial x^4\partial y^4} + 4\frac{\partial^8}{\partial x^2\partial y^6} + \frac{\partial^8}{\partial y^8}.$$

Expressions of shell displacements in terms of a potential function Φ_p $(p=1,2)$ are presented in the form:

$$u_p = -\frac{1}{E_p h_p}\frac{1}{R}\left(\nu_p\frac{\partial^3\Phi_p}{\partial x^3} - \frac{\partial^3\Phi_p}{\partial x\partial y^2}\right),$$

$$v_p = -\frac{1}{E_p h_p}\frac{1}{R}\left(\frac{\partial^3\Phi_p}{\partial y^3} + (2+\nu_p)\frac{\partial^3\Phi_p}{\partial x^2\partial y}\right), \tag{2}$$

$$w_p = \frac{1}{E_p h_p}\left(\frac{d^4\Phi_p}{dx^4} + \frac{d^4\Phi_p}{dy^4} + 2\frac{d^4\Phi_p}{dx^2 dy^2}\right).$$

The internal forces of the shell are determined by the formulas:

$$T_x^{(p)} = \frac{1}{R}\frac{\partial^4\Phi_p}{\partial x^2\partial y^2}, \qquad T_y^{(p)} = \frac{1}{R}\frac{\partial^4\Phi_p}{\partial x^4},$$

$$S^{(p)} = -\frac{1}{R}\frac{\partial^4\Phi_p}{\partial x^3\partial y}, \qquad H^{(p)} = -D^{(p)}(1-\nu_p)\frac{\partial^2 w_p}{\partial x\partial y}, \tag{3}$$

$$M_x^{(p)} = -D^{(p)}\left(\frac{\partial^2 w_p}{\partial x^2} + \nu_p\frac{\partial^2 w_p}{\partial y^2}\right),$$

$$M_y^{(p)} = -D^{(p)}\left(\frac{\partial^2 w_p}{\partial y^2} + \nu_p\frac{\partial^2 w_p}{\partial x^2}\right), \quad N_x^{(p)} = -D^{(p)}\left(\frac{\partial^3 w_p}{\partial x^3} + \frac{\partial^3 w_p}{\partial x\partial y^2}\right) = -D^{(p)}\frac{\partial}{\partial x}\left(\nabla^2 w_p\right),$$

$$N_y^{(p)} = -D^{(p)}\left(\frac{\partial^3 w_p}{\partial y^3} + \frac{\partial^3 w_p}{\partial x^2 \partial y}\right) = -D^{(p)}\frac{\partial}{\partial y}\left(\nabla^2 w_p\right).$$

The boundary conditions of the shell are written in the form:

- Hanging support on the sides $y=0$ and $y=b$

$$w_p = 0, \quad u_p = 0, \qquad T_2^{(p)} = 0, \quad M_2^{(p)} = 0, \qquad (p = 1, 2), \tag{4}$$

- symmetry on the line $x=0$

$$u_2 = 0, \quad \frac{\partial w_2}{\partial x} = 0, \ S^{(2)} + \frac{H^{(2)}}{R} = 0, \ N_1^{(2)} + \frac{\partial H^{(2)}}{\partial y} = 0, \tag{5}$$

- terminations on the side $x=L$

$$u_1 = 0, \quad v_1 = 0, \qquad w_1 = 0, \qquad \frac{\partial w_1}{\partial x} = 0, \tag{6}$$

- conjugation on the line $x=a$

$$u_1 = u_2, \ v_1 = v_2, \ \ w_1 = w_2, \ \frac{\partial w_1}{\partial x} = \frac{\partial w_2}{\partial x}, \ T_x^{(1)} = T_x^{(2)}, \ M_x^{(1)} = M_x^{(2)},$$

$$S^{(1)} + \frac{H^{(1)}}{R} = S^{(2)} + \frac{H^{(2)}}{R}, \ N_x^{(1)} + \frac{\partial H^{(1)}}{\partial y} = N_x^{(2)} + \frac{\partial H^{(2)}}{\partial y}. \tag{7}$$

Expanding the load function in a series:

$$q(y) = \sum_1^{\infty} q_m \sin\lambda_m y, \quad q_m = \frac{2}{b}\int_0^b q(y)\sin\lambda_m y dy, \quad \lambda_m = \frac{\pi m}{b},$$

solutions of equations (1) satisfying conditions (4) are represented in the form:

$$\Phi_p = \frac{E_p h_p}{D^{(p)}}\sum_{m=1}^{\infty}\frac{q_m}{\lambda_m^8} sin\,\lambda_m y + \sum_{m=1}^{\infty}\Phi_{pm}\, sin\,\lambda_m y, \ (p = 1, 2) \tag{8}$$

where

$$\Phi_{pm} = \sum_{j=1}^{8} C_{jm}^{(p)} e^{\alpha_{pj}\lambda_m x} = C_{1m}^{(p)} e^{\alpha_{p1}\lambda_m x} + C_{2m}^{(p)} e^{\alpha_{p2}\lambda_m x} + C_{3m}^{(p)} e^{\alpha_{p3}\lambda_m x} + C_{4m}^{(p)} e^{\alpha_{p4}\lambda_m x} +$$
$$+ C_{5m}^{(p)} e^{\alpha_{p5}\lambda_m x} + C_{6m}^{(p)} e^{\alpha_{p6}\lambda_m x} + C_{7m}^{(p)} e^{\alpha_{p7}\lambda_m x} + C_{8m}^{(p)} e^{\alpha_{p8}\lambda_m x} \tag{9}$$

The coefficients α_{pj} are the roots of the characteristic equation:

$$\left(\alpha^2 - 1\right)^4 + \frac{12(1-\nu^2)}{R^2 h^2 \lambda_m^4}\alpha^4 = 0. \tag{10}$$

Substituting (8) into the equation (2), for function w_p we get the following expressions:

$$w_p = \sum_{m=1}^{\infty}\frac{q_m}{\lambda_m^4 D^{(p)}} sin\,\lambda_m y + \sum_{m=1}^{\infty} w_{pm}\, sin\,\lambda_m y. \tag{11}$$

where: $$w_{pm} = \frac{1}{E_p h_p}\left(\frac{d^4\Phi_{pm}}{dx^4} + \lambda_m^4\Phi_{pm} - 2\lambda_m^2\frac{d^2\Phi_{pm}}{dx^2}\right).$$

By substituting (9) and (14) into conditions (4) - (7), the values of the coefficients $C_{jm}^{(p)}$ are determined after which all calculated values are determined by formulas (2) and (3).

Optimization of the shell according to the criterion of rigidity

Determination of the optimal parameters of the shell a, h_1, at which the greatest deflections of the structure, with a constant weight equal to the weight of the shell of constant thickness h_0, and given overall dimensions $\xi = 2L/b$, reach the smallest value, is reduced to the following nonlinear programming problem:

Find:

$$\min_{\bar{x}} \max_{p} w_p, \quad \bar{x} = \{a, h_1, h_2\}, \quad (p = 1,2), \tag{12}$$

with restrictions:

$$L(h_1 - h_0) = a(h_1 - h_2), \tag{13}$$

$$f/b \le 0.2, \ \ 0.01\delta_1 \le h_1 \le 0.2\delta_1, \ \ 0.01\delta_2 \le h_2 \le 0.2\delta_2 . \tag{14}$$

Here w_p - is the objective function, determined from (2), $\bar{x}$- is the control vector. Constraint (13) follows from the condition of the constancy of the weight of the structure of piecewise constant thickness, equal to the weight of the shell of constant thickness h_0. Restrictions (14) are due to the limits of applicability of the theory of thin shallow shells. Here f - is the lifting arrow of the shell, $\delta_1 = L - a$ when $L - a \le b$, $\delta_1 = b$ when $L - a \ge b$, $\delta_2 = 2a$ when $2a \le b$, $\delta_2 = b$ when $2a \ge b$.

Problem (16) - (18) is solved by the deformable polyhedron (DPM) method in combination with the direct search method [4].

Numerical calculations were performed for the case when a uniformly distributed load acts on the shell $q(y) = q_0 = Const$ when $\bar{h}_0 = h_0/b = 0.02$, $\xi = 2L/b$.

The optimal values of the parameters are calculated $\bar{a} = a/2L$, $\bar{h}_1 = h_1/b$, $\bar{h}_2 = h_2/b$, providing the smallest value of the greatest deflection $\max_p \bar{w}_p = w_p \dfrac{D_0}{q_k b^4}$, $(p = 1,2)$ panels, where $D_0 = \dfrac{E h^3}{12(1-\nu^2)}$, and the corresponding largest deflections in each of its sections. The results are shown in Table. In the same place, for comparison, the values of the reduced deflections $\bar{w}_0$ are given for an equilibrium plate of constant thickness h_0.

Table

Optimal parameters of a cylindrical panel

$\bar{h}_0$	ξ	a	$\bar{h}_1$	$\bar{h}_2$	$10^3 \cdot \bar{W}_1$	$10^2 \cdot \bar{W}_2$	$10^2 \cdot \bar{W}_0$
0.02	1.	0.01	0.01635	0.19885	0.04358	0.03108	0.1522
0.02	2.	0.005	0.0182	0.1982	0.22244	0.06216	0.6475
0.02	3.	0.005	0.01855	0.1635	0.50745	0.09910	1.0331
0.02	5.	0.005	0.0192	0.09925	1.01120	0.28409	1.2953
0.02	10.	0.005	0.01995	0.02495	1.31564	1.23127	1.3171

Conclusion

As a result, compared to a platform of constant thickness, the largest reduction in bending is about 3.5 times.

Calculations were performed to find the optimal values of the parameters providing the smallest values of maximum bending.

Also, as a result of solving the problem by different methods, the results are matched with hundredth accuracy.

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ԻՏԵՐԱՏԻՎ ՔԱՅԼԱՅԻՆ ՆՎԱԶԱՐԿՄԱՆ ՄԵԹՈԴԻ ԿԻՐԱՌՈՒՄԸ ԳԼԱՆԱՁԵՎ ՀԱՐԹԱԿԻ ՕՊՏԻՄԱԼ ՆԱԽԱԳԾՄԱՆ ԽՆԴՐԻ ԼՈՒԾՄԱՆ ՀԱՄԱՐ, ՎԵՐՋԻՆԻՍ ՃԿՄԱՆ ՊԱՅՄԱՆՆԵՐՈՒՄ

Ա.Գ. Պողոսյան, Դ.Ս. Բաղասյան, Ա.Տ. Սիմոնյան
Հայաստանի ազգային պոլիտեխնիկական համալսարան

Առաջարկվում է օպտիմալացման խնդրի լուծման այնպիսի մի մոդել, որը թույլ է տալիս հավասարության և անհավասարության տեսքով սահմանափակումների դեպքում կիրառել Ներլդեր-Միդի իտերատիվ քայլային նվազարկման մեթոդը: Այս մոդելի կիրառումը ներկայացվում է իզոտրոպ նյութից պատրաստված գլանաձև վահանակի, ճկման ընթացքում օպտիմալ նախագծման օրինակով: Որոշվում են, հարթակի երկրաչափական պարամետրերի օպտիմալ արժեքները, որոնց դեպքում առաջադրված են եզրաչափային չափերը և ապահովված է նվազագույն ճկում: Ինչպես նաև կատարվում է արդյունքների համեմատում, դրանց ճշտության լրացուցիչ ստուգման նպատակով, վերջիններիս համեմատման համար, խնդիրը լուծվում է նաև ուղիղ որոնման մեթոդով:

Բանալի բառեր. նվազարկում, առավելարկում, իզոտրոպ նյութ, բեռ, հարթակ, ճկում, իտերացիա:

ПРИМЕНЕНИЕ МЕТОДА ИТЕРАТИВНОГО ПОШАГОВОГО УМЕНЬШЕНИЯ ДЛЯ РЕШЕНИЯ ЗАДАЧИ ОПТИМАЛЬНОГО ПРОЕКТИРОВАНИЯ ЦИЛИНДРИЧЕСКОЙ ПЛАТФОРМЫ ПРИ ЕЕ ИЗГИБЕ

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Предлагается модель решения задачи оптимизации, которая позволяет использовать метод итерационно инкрементной минимизации Нелдера-Мида в случае наличия ограничений как в виде равенств, так и неравенств. Применение данной модели иллюстрируется на примере оптимального проектирования цилиндрической панели изготовленной из изотропного материала, при изгибе. Определяются оптимальные значения геометрических параметров панели, обеспечивающие минимальное значение прогиба при постоянном весе, равном весу панели постоянной толщины, и ее заданных габаритных размерах. Результаты также сравниваются, с целью дальнейшей проверки их точности, для сравнения последних, проблема также решается методом прямого поиска.

Ключевые слова: оптимизация, максимизация, изотоп материал, нагрузка, панель, изгиб, итерационный.

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