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ON CALCULATION OF HYDRAULIC FRICTION LOSSES

N.M. Sargsyan[1], A.A. Sarukhanyan[2]
[1]*Vanadzor State University, Armenia*
[2]*National University of Architecture and Construction of Armenia*

An analytic calculation method for energy loss in a straight hydraulically smooth pipe of uniform cross-section through which a viscous fluid flows in steady-state conditions has been suggested. This method enables to determine hydraulic friction losses based on a unified approach, i.e. without binding it to the fluid flow regime. Satisfactory convergence of calculated and known in literature values of friction resistance coefficients has been shown.

Key words: *fluid, viscosity, velocity, energy, loss, pressure*.

Introduction

Energy losses initiated in straight hydraulically smooth pipes of uniform cross-section, usually, in hydraulics are calculated by Darcy-Weisbach empirical formula [1].

$$p = \lambda \cdot \frac{l}{d} \cdot \rho \frac{\langle u \rangle^2}{2} \tag{1}$$

where $\langle u \rangle$ is average velocity of the flow, l is the length of the pipe, d is the diameter of the pipe, ρ is the fluid density, λ is the hydraulic friction coefficient.

The value of dimensionless coefficient λ in case of a circular pipe can be determined after laying down a law on velocity distribution u through the cross-section of the pipe. In laminar flow of fluid the value λ is determined theoretically and in turbulent flow it is determined by empirical or semi-empirical formulae.

For hydraulically smooth pipes current calculation formulae for λ stipulate Reynolds number dependence of this coefficient.

Due to formal assumption on existence of λ=f(Re) relationship in Eq.(1), an erroneous conclusion is drawn on proportionality between pressure loss and flow rate for various degrees (Δp~⟨u⟩, Δp~⟨u⟩1.75 etc.), under different regimes of fluid flow. Such an assumption contradicts requirement of the general theory of dynamics on mechanical system's kinetic energy change.

The present study suggests a new method for calculation of hydraulic friction losses in flow of viscous fluid in a horizontal circular pipe under isothermic and steady-state conditions.

Singularity of the suggested method lies in that that made hydrodynamic calculations do not have any connection with the stream flow regime.

Detailed description of the mechanism of the fluid flow in a pipe, as we think of it, is presented in [2,3]. Here we present results, necessary for calculation of hydraulic losses in a pipe.

Scientists of many countries worked at issues of the movement of suspended particles in a turbulent stream. The principle difficulty here lies in the fact that different models of turbulence are not sufficiently perfect and up to now are subjects of theoretical and experimental study. As far back as in the middle of the last century academician A.N. Kolmogorov carried out study of the issue, the main ideas on locally isotropic turbulence of the stream being underlay in a so-cold $k-\varepsilon$ model of turbulence [1]. Later his hypotheses were used for closure of equations describing the movement of temperature heterogeneous streams [2] and suspended particles in turbulent stream [3,4]. In his fundamental works F.I.Frankl formulated equations of momentum and energy of turbulent pulsation fluid and hard components of flow, taking into account interaction between hard particles and fluid [5,6].Issues concerning the force of interaction between hard particles and fluid in their pulsating movement were also considered by J.O.Hinze [7,8]. Mud streams are formed as a result of interaction of a number of natural factors.

Initial section

The flow at the initial section is divided into central (potential) and peripheral (viscous) zones. In the central zone the viscosity does not take into consideration, and the flow velocity is constant across the pipe section and directed along its axis.

At the expense of the boundary layer on the wall of a pipe the diameter of the central zone gradually decreases and the velocity increases.

Within potential flow according to Bernoulli equation

$$\frac{U_e^2 - U_-^2}{2} = \frac{p_- - p}{\rho} , \tag{2}$$

where $U_- = \langle u \rangle, p_- -$ is velocity and pressure at the pipe entry, $U_c, p -$ velocity and pressure at an arbitrary distance z along the pipe. For the given flow of the fluid the length of the initial section $l_0 = const$, and the corresponding velocity at the end of that section is $U_c = U_0$.
If divide both sides of Eq. (2) on z, we get

$$\frac{U_e^2 - U_-^2}{2z} = \frac{U^2}{2z} = \frac{p_- - p}{\rho z} \tag{3}$$

The value $\dfrac{p_- - p}{\rho z} = a$ has a meaning of acceleration and U is determined from Eq.(3)

$$U = \sqrt{2az} \tag{4}$$

A peculiarity of the boundary layer, developed on the surface of the initial section of the pipe under pressure flow of fluid, is in that that it is formed when the external flow is accelerated.

To carry out analytical study of fluid flow within boundary layer equations of continuity and stream movement are used, which in plane orthogonal coordinates have the following form [2]

$$\frac{\partial u}{\partial z} + \frac{\partial \upsilon}{\partial r} = 0 \tag{5}$$

$$\rho\left(u\frac{\partial u}{\partial z} + \upsilon\frac{\partial u}{\partial r}\right) = \rho a + \mu\frac{\partial^2 u}{\partial r^2} , \tag{6}$$

including, if the conditions are assumed $\delta_e << R, \dfrac{\partial^2 u}{\partial z^2} << \dfrac{\partial^2 u}{\partial r^2}$.

As long as the width of the boundary layer δ_c is small as compared to the pipe radius R the layer can be considered as plane [3].

In Eqs.(5) and (6) $r -$ is the distance to the axis of the pipe, u, υ are velocity components along z and r, $\mu = \rho \cdot \nu$ is coefficient of dynamic viscosity.

Solution of Eqs.(5) and (6) should be carried out under boundary conditions

$$u = \upsilon = 0 \text{ if } r = 0$$

$$u = U_c \text{ if } r \to \delta_c \tag{7}$$

$$u = U_{-} = \langle u \rangle \text{ if } z = 0$$

Through the use of the below dimensionless functions

$$\eta = \frac{1}{\sqrt{2}} \cdot \left(\frac{2a}{\upsilon^2} \right)^{1/4} \cdot \frac{1}{z^{1/4}} \tag{8}$$

$$f = \frac{1}{\sqrt{2}} \cdot \frac{\psi(z,r)}{\left(\frac{2a}{\upsilon^2} \right)^{1/4} \cdot \upsilon \cdot z^{3/4}} \tag{9}$$

input equation (6) can be written as

$$f''' + \frac{3}{2} f \cdot f'' - (f')^2 + 1 = 0 \ , \tag{10}$$

Boundary conditions (7) in new variables have the following view

$$f = 0, \quad f' = 0 \text{ if } \eta = 0$$

$$f' = 1 \text{ if } \eta = \eta_c$$

For small values $\mu \to 0$ approximate analytical solution of Eq.(10), we have

$$f = \frac{f_0''}{2!} \cdot \eta^2 - \frac{1}{3!} + \frac{f_0''}{2 \cdot 5!} - \cdots \ , \tag{11}$$

where $f_0'' = 1.272$ according to the numerical solution of Eq.(10).

Judging by the order of members values, in solution (11) the following approximation can be assumed – if $\eta \le 0.3$ confine to the first member (absolute error is $\Delta = Rn(0.3) = 4.5 \cdot 10^{-3}$) and if $0.3 < \eta < 1.2$ confine to lowest orders of η (first two members $\Delta = Rn(1.2) = 0.013$).

The performed analysis enables reveal three regions in the boundary layer, where velocity change along the radius occurs by various regularities [2]

First region: viscous sublayer, $0 < \eta < 0.3$

$$f_I' = f_0' \cdot \eta \tag{12}$$

Second region: interlayer or buffer layer $0.3 < \eta < 1.2$

$$f_{II}' = 0.675 \cdot \eta^{0.5} \tag{13}$$

Third region: $1.2 < \eta < 3$

$$f_{III}' = 1 - 0.922 \frac{e^{-\frac{3}{4}\eta^2}}{\eta} \tag{14}$$

The condition $f' = u/U \approx 0.9997$ is fulfilled approximately when $\eta = 3$.

The average velocity of flow $\langle u \rangle$ in cross-section at l_0 distance from the pipe entry is determined by the below equation

$$\langle u \rangle = \frac{2\pi\left(\int_0^{r_1} urdr + \int_{r_1}^{r_2} urdr + \int_{r_2}^{r_3} urdr\right)}{\pi R^2} = \frac{2U_0}{\eta_e^2}\left(\int_0^{\eta_1} f_I'\eta d\eta + \int_{\eta_1}^{\eta_2} f_{II}'\eta d\eta + \int_{\eta_2}^{\eta_3} f_{III}'\eta d\eta\right)$$

After performing corresponding calculations, we get

$\langle u \rangle = 0.64U_0$

If the length l_0 of the initial section express through the diameter of the pipe and average velocity, according to Eq.(8) we have

$$l_0 = 0.017d\,\mathrm{Re} \tag{15}$$

For the given fluid flow acceleration of potential flow according to Eq.(2) is determined by

$$a = \frac{u_0^2 - \langle u \rangle^2}{2l_0} = 1.44\frac{\langle u \rangle}{2l_0} \tag{16}$$

It is expedient to derive a reasonable formula of resistance on the basis of the general theorem of dynamics of kinetic energy change of a system, on the basis of the following simple considerations.

At the time of fluid flow through the pipe the profile of velocities undergo a continuous change, redistribution of the mechanical energy of the flow occurs.

Real kinetic energy of some m mass of the fluid passing during dt time through water cross-section under study, greater than the average kinetic energy (calculated on the basis of the average velocity $< u >$)

The profile of velocity u in the cross-section of the boundary layer is characterized by irregular distribution: its value continuously is changed from zero to $U_e = \sqrt{2al + (< u >)^2}$.

Let us single out a section at the entry of the pipe of l length and consider two versions of fluid flow in that. The fluid of mass m flowing at U_e velocity, reaching at the end of the section l, could receive kinetic energy $T_1 = \frac{mU_e^2}{2}$, and when flow at velocity $< u > - T_2 = m\frac{(< u >)^2}{2}$.

The quantity of kinetic energy loss $\Delta T = T_1 - T_2$, conditioned by the friction force, can be determined using the theory of dynamics on kinetic energy change

$$\frac{\pi d^2}{4} \cdot \rho \cdot \frac{U_e^2 - \langle u \rangle^2}{2} = \frac{\pi d^2}{4} \cdot \Delta p - \tau \cdot \pi dl$$

from which pressure fall at l distance can be found

$$\Delta p = 4\tau \frac{l}{d} + \rho \left(\frac{U_e^2}{2} - \frac{\langle u \rangle^2}{2} \right) \tag{17}$$

To a first approximation $\tau = \tau_w$ can be accepted, where τ_w - is the average shearing stress on the wall of the pipe, where the velocity gradient achieves the maximum value.

Taking into account the below equation

$$u = \frac{\partial \psi(r,z)}{\partial r} = \frac{\partial \psi}{\partial \eta} \cdot \frac{\partial \eta}{\partial r} = \left(\frac{2a}{\upsilon^2} \right)^{1/2} \cdot \upsilon \cdot z^{1/2} \cdot f',$$

the local shear stress on the surface of a hard wall is determined

$$\tau_0(z) = \mu \left. \frac{\partial u}{\partial r} \right|_{r=0} = \frac{1}{\sqrt{2}} \mu \cdot f_0'' \cdot \left(\frac{2a}{\upsilon^2} \right)^{3/4} \cdot \upsilon \cdot z^{1/4},$$

and the average value of shear stresses is defined by the below equation

$$\tau_w = \frac{4}{5\sqrt{2}} \cdot f_0'' \cdot \mu \cdot \left(\frac{2a}{\upsilon^2} \right)^{3/4} \cdot \upsilon \cdot l^{1/4}$$

Substituting τ_w in Eq.(17), we get

$$\Delta p_1 = 2.88 \rho \upsilon^2 \left(\frac{2al}{\upsilon^2} \right)^{3/4} \cdot \frac{l^{1/2}}{d} + \rho \left(\frac{U_e^2 - \langle u \rangle^2}{2} \right) = 2.88 \rho \cdot 2al \frac{l^{1/2}}{d \cdot \left(\frac{2al}{\upsilon^2} \right)^{1/4}},$$

or

$$\Delta p_1 = 5.76 \frac{\rho U^2}{2} \cdot \frac{1}{U^{1/2}} \cdot \frac{\upsilon^{1/2} \cdot l^{1/2}}{d} + \frac{\rho U^2}{2} \tag{18}$$

With the help of formulas (4) and (16) Eq.(18) after simple transformations can be presented in the following view

$$\Delta p_1 = \left[7.6 \frac{1}{\mathrm{Re}^{1/2}} \left(\frac{l}{l_0} \right)^{3/4} \left(\frac{l}{d} \right)^{1/2} + 1.44 \frac{l}{l_0} \right] \rho \frac{\langle u \rangle^2}{2} = \zeta \frac{\rho \langle u \rangle^2}{2}, \tag{19}$$

where ζ is a coefficient of resistance.

At the end of the initial section $l = l_0$ pressure fall will be

$$\Delta p_0 = 2.42 \rho \frac{\langle u \rangle^2}{2}$$

Hydraulic losses at the initial section according to G.Shlichting is determined by

$$\Delta p_0 = 1.16 \frac{\langle u \rangle^2}{2}$$

Stabilized flow: $l > l_0$

It is assumed that velocity profile formed at the end of the initial section, in further flow of fluid remains unchanged.

Under these conditions velocity distribution function is presented as

$$u = \left(\frac{2a}{\upsilon^2}\right)^{1/2} \upsilon l_0^{1/2} f' = U_0 f'$$

The local shear stress is determined by the below equation

$$\tau_0(z) = \mu \frac{\partial u}{\partial r}\bigg|_{r=0} = \frac{1}{\sqrt{2}} f_0'' U_0 \mu \left(\frac{2a}{\upsilon^2}\right)^{1/4} \frac{1}{z^{1/4}}$$

The average value of shearing stresses is determined by the below formula

$$\tau_w = \frac{4}{3\sqrt{2}} f_0'' U_0 \mu \left(\frac{2a}{\upsilon^2}\right)^{1/4} \frac{1}{l^{1/4}}$$

Pressure loss in the l long section of under consideration will be

$$\Delta p_2 = \frac{16}{3\sqrt{2}} f_0'' U_0 \mu \left(\frac{2a}{\upsilon^2}\right)^{1/4} \frac{l^{1/4}}{d}$$

Making some simple transformations of the above equation, we have

$$\Delta p_2 = 45.5 \cdot \frac{1}{\mathrm{Re}^{3/4}} \cdot \left(\frac{l}{d}\right)^{3/4} \cdot \frac{\rho \langle u \rangle^2}{2}$$

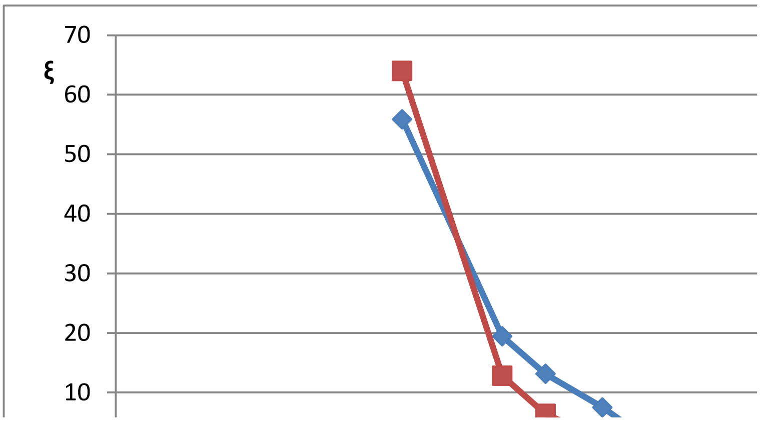

Figure 1. Comparison of friction resistance coefficients for $l/d = 100$

1- theoretical curve plotted by Eq.(20) $\zeta = \left[\frac{45.5}{\mathrm{Re}^{3/4}} \cdot \left(\frac{l}{d}\right)^{3/4} + 2.42\right]$, 2- plotted by $\zeta = \frac{64}{\mathrm{Re}} \cdot \frac{l}{d}$ [1],

3- plotted by $\zeta = \frac{0.316}{\mathrm{Re}^{0.25}} \cdot \frac{l}{d}$ [1], 4- plotted by $\zeta = \left(0.0032 + \frac{0.221}{\mathrm{Re}^{0.237}}\right) \cdot \frac{l}{d}$ [4].

Pressure general loss in the pipe is summed up from losses at the initial section where velocity profile is stabilized and in the section of stabilized flow

$$\Delta p = \Delta p_2 + \Delta p_0 = \left[45.5 \cdot \frac{1}{\mathrm{Re}^{3/4}} \cdot \left(\frac{l}{d}\right)^{3/4} + 2.42\right] \frac{\rho \langle u \rangle^2}{2} \qquad (20)$$

Comparison of calculated values of the friction resistance coefficient with experimental data available in the literature (see the Figure)

Discrepancy between the two Averaged 8 per cent.

Conclusion

A new method is suggested for calculation of hydraulic friction loss along a pipe of constant cross-section without binding it to the fluid flow regime.

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Principal legend

l – length of the pipe, m, l_0 – length of the initial section, m, r – radius, m, d – diameter of the pipe, m, p – pressure, Pa, $f(\eta)$ – dimensionless function of the stream, u, υ – longitudinal and transverse components of velocity within viscous flow, m/s, $\langle u \rangle$ – average velocity of flow, m/s, U_0 – velocity of fluid particles on the pipe’s axis achieved at the end of the initial section, m/s, U_c - velocity at the boundary layer edge, m/s, R– radius of the pipe, m, ψ – function of the flow, m^2/s, δ_e – thickness of the film, m, η_e – dimensionless thickness of the film, ρ – fluid density, kg/m^3, ζ - coefficient of friction resistance, μ – coefficient of dynamic viscosity, Pa s, υ – kinematic viscosity of fluid, m^2/s, τ_0 – local shear stress, Pa, τ_w - average value of shearing stresses, Pa, $\mathrm{Re} = \frac{\langle u \rangle d}{\nu}$ – Reynolds criterion.

ՇՓՄԱՆ ՀԻԴՐԱՎԼԻԿԱԿԱՆ ԿՈՐՈՒՍՏՆԵՐԻ ՄԱՍԻՆ

Ն.Մ. Սարգսյան[1], Ա.Ա. Սարուխանյան[2]
[1]Վանաձորի պետական համալսարան
[2]Ճարտարապետության և շինարարության Հայաստանի ազգային համալսարան

Առաջարկվում է հորիզոնական, հիդրավլիկորեն ողորկ, հաստատուն տրամագծի խողովակներում մածուցիկ հեղուկի ստացիոնար շարժման պայմաններում էներգիայի կորստի հաշվարկային բանաձև: Էներգիայի կորստի հաշվարկի առաջարկվող մեթոդաբանության հիմքում դրված է ընդհանուր տրամաբանություն, որն ընդունելի է դիմադրության բոլոր գոտիների համար և կախված չէ շարժման ռեժիմից: Կատարվել են համեմատական վերլուծություններ առաջարկվող բանաձևի և գրականության մեջ հայտնի բանաձևերի արդյունքների միջև:

Բանալի բառեր. հեղուկ, մածուցիկություն, արագություն, էներգիա, կորուստ, ճնշում:

К РАСЧЕТУ ГИДРАВЛИЧЕСКИХ ПОТЕРЬ НА ТРЕНИЕ

Н.М. Саргсян[1], А.А. Саруханян[2]
[1]Ванадзорский государственный университет
[2]Национальный университет архитектуры и строительства Армении

Предлагается аналитический метод расчета потери энергии в прямой, гидравлически гладкой трубе постоянного сечения при течении вязкой жидкости в стационарных условиях. Предлагаемая методика позволяет определить гидравлические потери на трение из единой позиции, т.е. без привязки к режиму течения жидкости. Показана удовлетворительная сходимость расчетных и известных в литературе значений коэффициентов сопротивлений на трение.

Ключевые слова: жидкость, вязкость, скорость, энергия, потери, давление.